Appendix C
Spectrum Matching

C.1 Spectrum Matching Process

a. This appendix discusses the generation of realistic acceleration time-histories whose response spectra closely match a smooth design response spectrum. The spectrum-matching procedure begins with an acceleration time-history whose characteristics reasonably represent the ground motions expected for the site. (Guidelines for selecting such an initial time-history are discussed in paragraph 5.3.) This initial time-history has individual spectral peaks and valleys that deviate from the smooth design spectrum. The objective of the matching procedure is to reduce these deviations in the period range important to the structure while preserving the nonstationary characteristics (which are critical to nonlinear analysis) of the initial time-history as much as possible. The design spectral values are generally given at a fine grid of periods so that the response spectrum from the matched time-history is also smooth at the intervening periods (paragraph 5.5*f*). Convergence with the design spectrum is achieved when spectral values of the modified time-history are within specified deviations from the design values at every period being matched (paragraph 5.5*c*).

b. Spectrum matching is conducted by adding (subtracting) elementary wavelets to (from) the initial acceleration time-history. Each wavelet is intended to match the response spectrum at one period. Matching is performed either period by period or simultaneously for a group of periods; the latter usually provides a better convergence property, and it is adopted in this study. Spectral convergence typically requires several iterations of matching. Excessive iterations should be avoided to assure the preservation of nonstationary characteristics of the initial time-history. The existing matching procedures can be divided into two categories.

(1) Time-Domain Approach (Lilhanand and Tseng 1988; Abrahamson 1992). Matching is accomplished by adding (subtracting) finite-duration wavelets to (from) the initial time-history. This approach normally provides a close fit to the target. One representative wavelet is the impulse response time-history of the single-degree-of-freedom oscillator reversed in time. This wavelet abruptly ceases (i.e., becomes zero) after the peak-response time and thus limits the temporal extent of the modification made to the time-history. Another commonly used wavelet is the tapered sinusoid with the level of tapering being period dependent. Timing of these wavelets is selected so that it is in phase with the peak response of the acceleration time-history. Detailed descriptions of the mathematical formulation and the numerical algorithms are given in Lilhanand and Tseng (1988) and Abrahamson (1992).

(2) Frequency-Domain Approach (Gasparini and Vanmarcke 1976; Silva and Lee 1987; Bolt and Gregor 1993). The frequency-domain matching adjusts only the Fourier amplitudes while the Fourier phases are kept unchanged. This procedure is equivalent to adding or subtracting sinusoids (with the Fourier phases of the initial time-history) in the time domain and does not always provide as close a fit to the design spectrum as the time-domain approach. Because the sinusoids extend the entire length of the time-history, the spectrum-compatible motion produced by this procedure shows greater visual differences from the initial time-history than does the time-domain approach.

c. To preserve the nonstationary characteristics of the initial time-history, it is essential to start with an acceleration time-history whose spectrum is as close to the target spectrum as possible in the period range of interest. A close initial fit also ensures a speedy convergence to the design values. In many cases, a good initial match can be obtained by simply applying a single scale factor to the initial time-history. After spectrum matching, the following checks on the resulting acceleration time-history are recommended in addition to checking the spectrum match:

(1) A check should be made for baseline drift in the final time-history. The baseline should be corrected, and the resulting baseline-corrected acceleration time-history should be checked again for spectrum compatibility, then rematched if necessary. Baseline correction typically has little effect on the spectral values in the period ranges of interest.

(2) The acceleration, velocity, and displacement time-histories should be examined to ensure that they are reasonably close to the original or target values in terms of peak values, wave form, strong shaking duration, and other critical features such as the near-fault velocity pulse (fling).

(3) Power spectral density (PSD) functions should be examined to ensure a broad distribution of energy in the final spectrum-compatible motion as a function of Fourier period. There should not be significant deficiencies in the energy at periods important to the structure.

C.2 Examples

a. A target spectrum was specified for 5 percent damping at 220 periods ranging from 0.03 to 2 sec. Ten horizontal acceleration time-histories were selected and matched to the target spectrum using both the time-domain and frequency-domain approaches. The spectral value at the 0.03-sec period was taken as equal to the peak ground acceleration (PGA) value. For these examples, multiple-damping design spectra were not specified. Before matching was done, each record was scaled so that the initial overall fit to the target spectrum was improved. The scale factors, along with other information on the selected records, are given in Table C-1. All of the selected time-histories were recorded close to faults.

b. Computer codes RSPMATCH (Abrahamson 1992) and RASCAL (Silva and Lee 1987) were used to perform time-domain and frequency-domain matching, respectively. For the time-domain matching, the target spectra were extended to a 10-sec period by setting spectral values of the initial scaled time-histories as target values in the period range of 2 to 10 sec. This extension is needed to prevent the matching procedure from unduly modifying the motions at periods longer than 2 sec, which dominate the velocity and displacement time-histories. The number of iterations was restricted to be less than 10. The impulse response of a single-degree-of-freedom system (Lilhanand and Tseng 1988) was used as the adjustment wavelet. The resulting mismatch to the design spectral values was generally less than 10 percent.

c. Baseline-corrected acceleration, velocity, and displacement time-histories for the scaled initial time-histories, the frequency-domain spectrum-matched time-histories, and the time-domain spectrum-matched time-histories are shown in Figures C-1 to C-10. Also shown in these figures are the modifications to the initial time-history. Response spectra are shown in the top plot of Figures C-11 through C-20. The bottom plot of these figures shows the smoothed power spectral density functions. The power spectral density function *PSD(T)* of an acceleration time-history is estimated as

$$PSD(\omega) = \frac{2\,|\,F(\omega)\,|^2}{2\pi T_m} \tag{C-1}$$

where

T = Fourier frequency

|F(T)| = Fourier amplitude computed over a time window which contains acceleration that is near maximum and near stationary power

T_m = duration of the time window

The time window used for each record is shown as dashed lines in Figures C-1 to C-10. To remove the irregularity, the smoothed *PSD(T)* is then obtained by computing the average value over the frequency range from $T - 0.2T$ to $T + 0.2T$.

The following observations on the spectrally matched records are made:

(1) Both matching procedures produce an adequate fit to the target spectra. However, in general, the time-domain matching produces a closer fit than the frequency-domain matching does.

(2) The spectrum-compatible acceleration time-histories are reasonably similar to the initial acceleration time-histories. Modifications introduced by the two matching procedures show quite different time characteristics. The time-domain modification typically has a shorter time span.

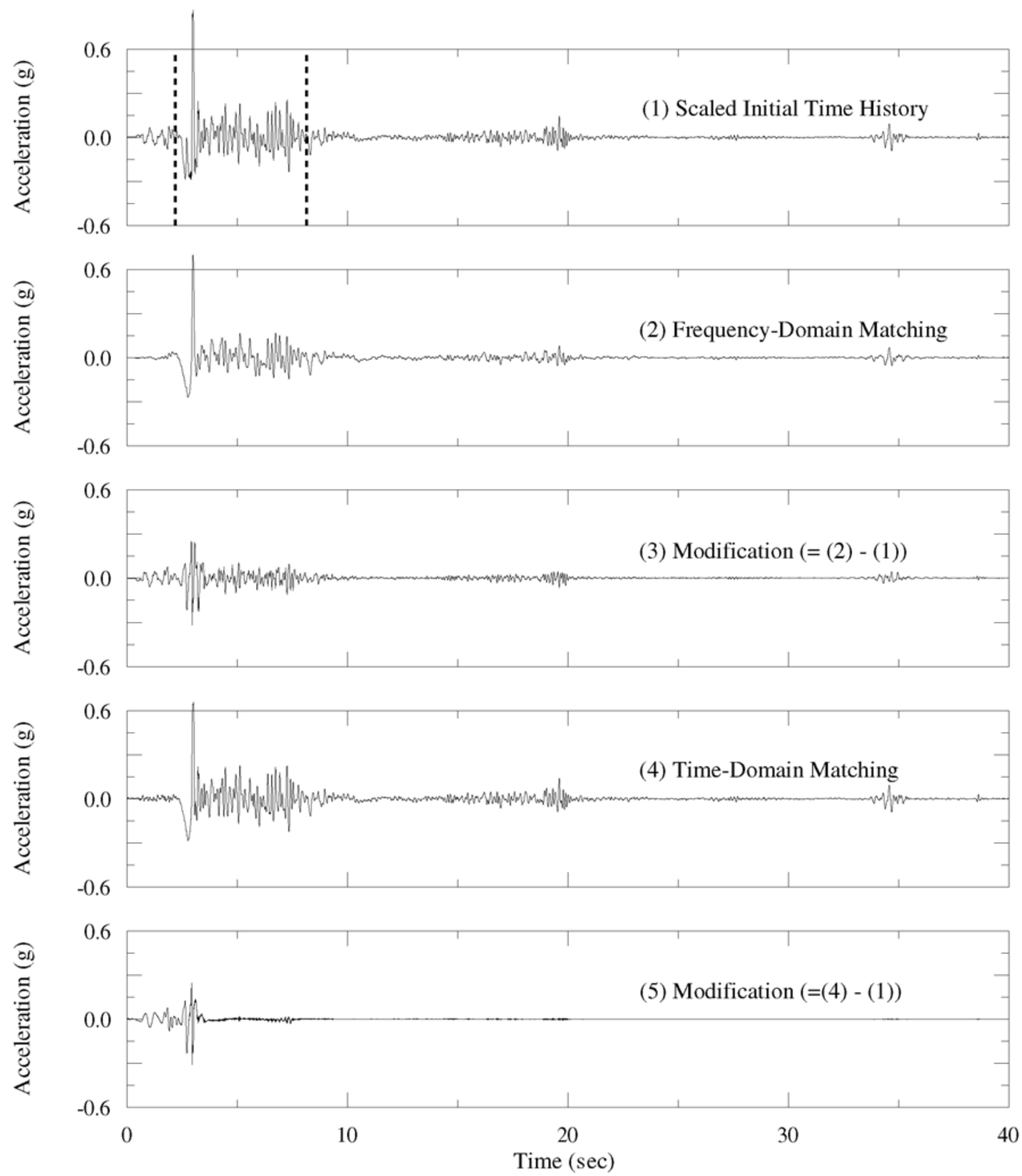
(3) Because matching is performed up to a 10-sec period in the time-domain approach, modification of spectral values at periods longer than the longest period of the target spectra is introduced occasionally. This undesired modification is quite obvious in some of the velocity and displacement time-histories shown in Figures C-1 to C-10, and it can be easily removed by high-pass filtering the spectrum-compatible acceleration time-histories. This is not a problem for the frequency-domain approach. It does not have this problem because it does not alter Fourier amplitudes beyond the longest period of the target spectra.

(4) There is no apparent energy deficiency in the matched records (Figures C-11 to C-20). The matching procedure reduces energy at periods where initial spectra are higher than the target spectra and increases energy at periods where initial spectra are lower.

(5) Both matching procedures are unable to simultaneously fit the PGA and short-period response of the Petrolia record from the 1992 Cape Mendocino earthquake (Figure C-11), despite an almost perfect fit to the target at longer periods. This inability to match is due to the coupling of short-period responses via the large acceleration pulse near the 3-sec time mark (Figure C-1a). This pulse controls the PGA and short-period spectral response (0.03 sec (PGA) to 0.3 sec), and it will not support a lower PGA value while matching the target from the 0.08- to 0.3-sec period. Even though this type of spectral coupling does not occur in the other nine histories used in this appendix, it is encountered occasionally. Special effort is usually needed to break up the spectral coupling.

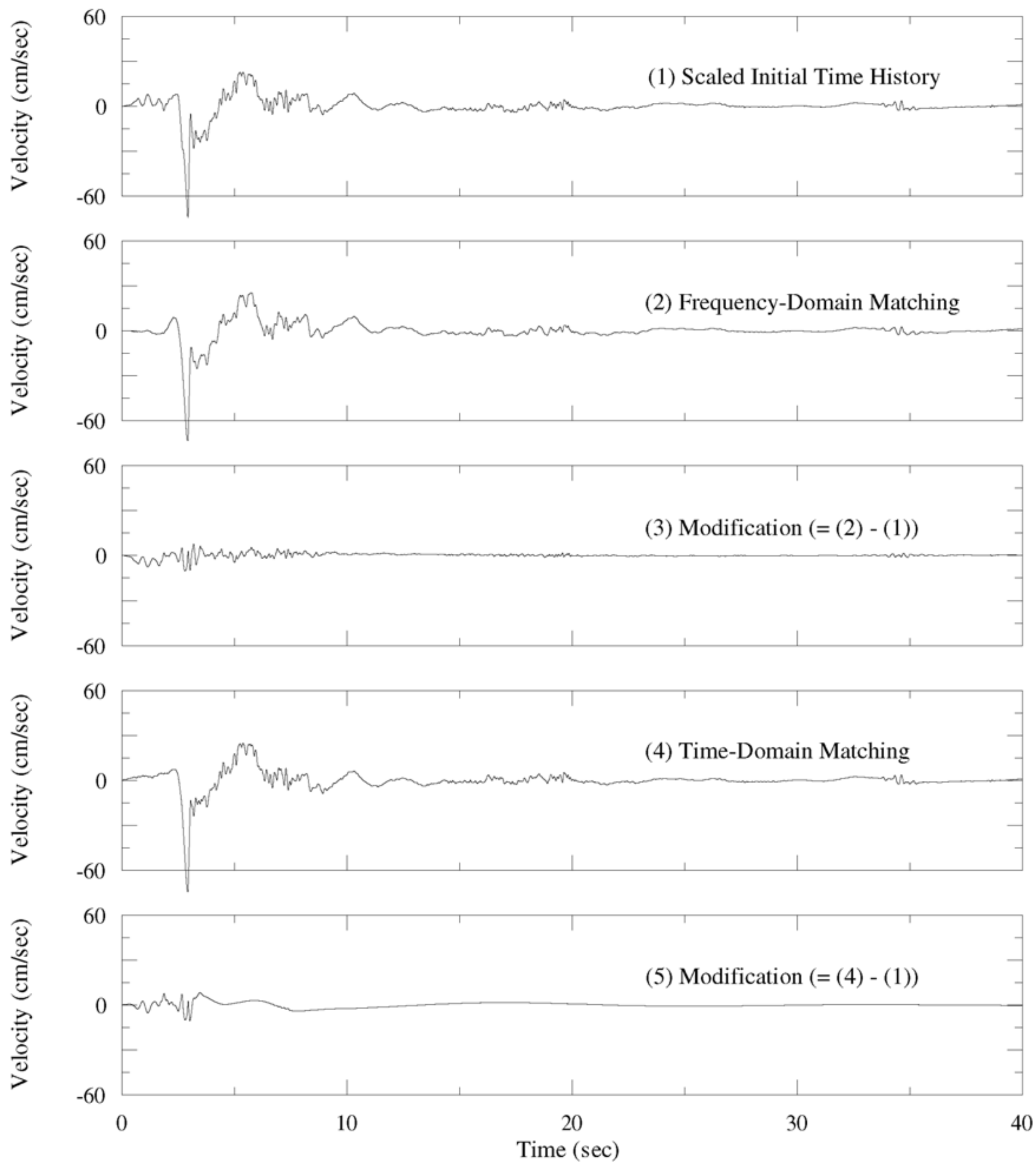
Table C-1
Acceleration Time-Histories Selected for Spectrum Matching

Record	Earthquake	M_W	Closest Distance, km	Component deg	Scale Factor
Petrolia	1992 Cape Mendocino	7.1	8.5	000	0.58
Gilroy	1989 Loma Prieta	6.9	11.6	067	1.43
Griffith Park	1971 San Fernando	6.6	17.4	270	2.85
Halls Valley	1984 Morgan Hill	6.2	3.4	240	2.22
Gazli	1976 Gazli	6.8	3.0	090	0.70
Pacoima	1994 Northridge	6.7	9.8	265	1.34
Coyote Lake Dam	1984 Morgan Hill	6.2	0.1	195	0.66
SCSE	1994 Northridge	6.7	7.8	018	0.85
Pacoima Dam	1971 San Fernando	6.6	2.8	254	0.60
UCSC BRAN	1989 Loma Prieta	6.9	10.3	090	0.83



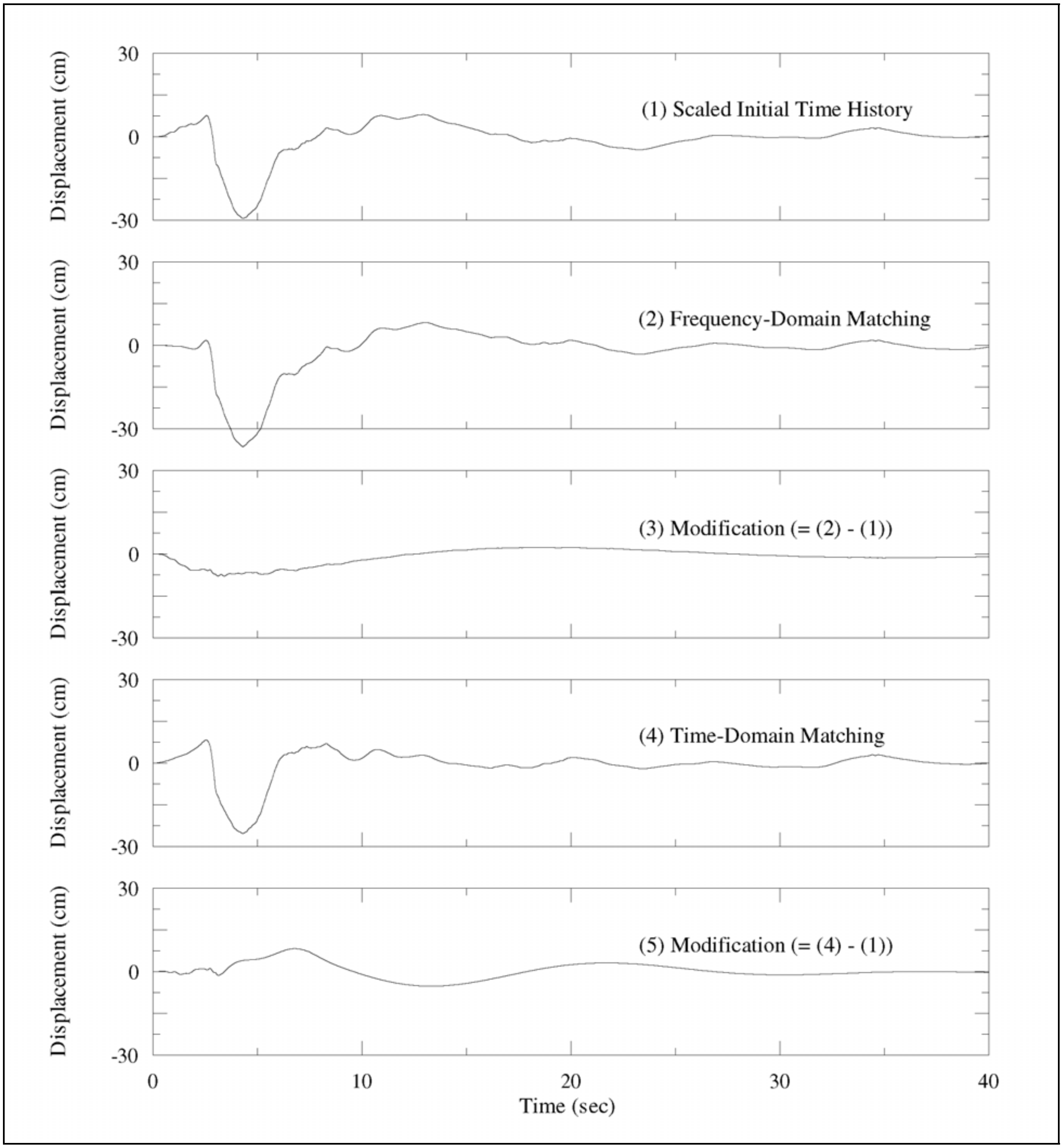
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-1. Comparison of time-histories for Petrolia recording (component 000E), 1992 Cape Mendocino earthquake (Sheet 1 of 3)



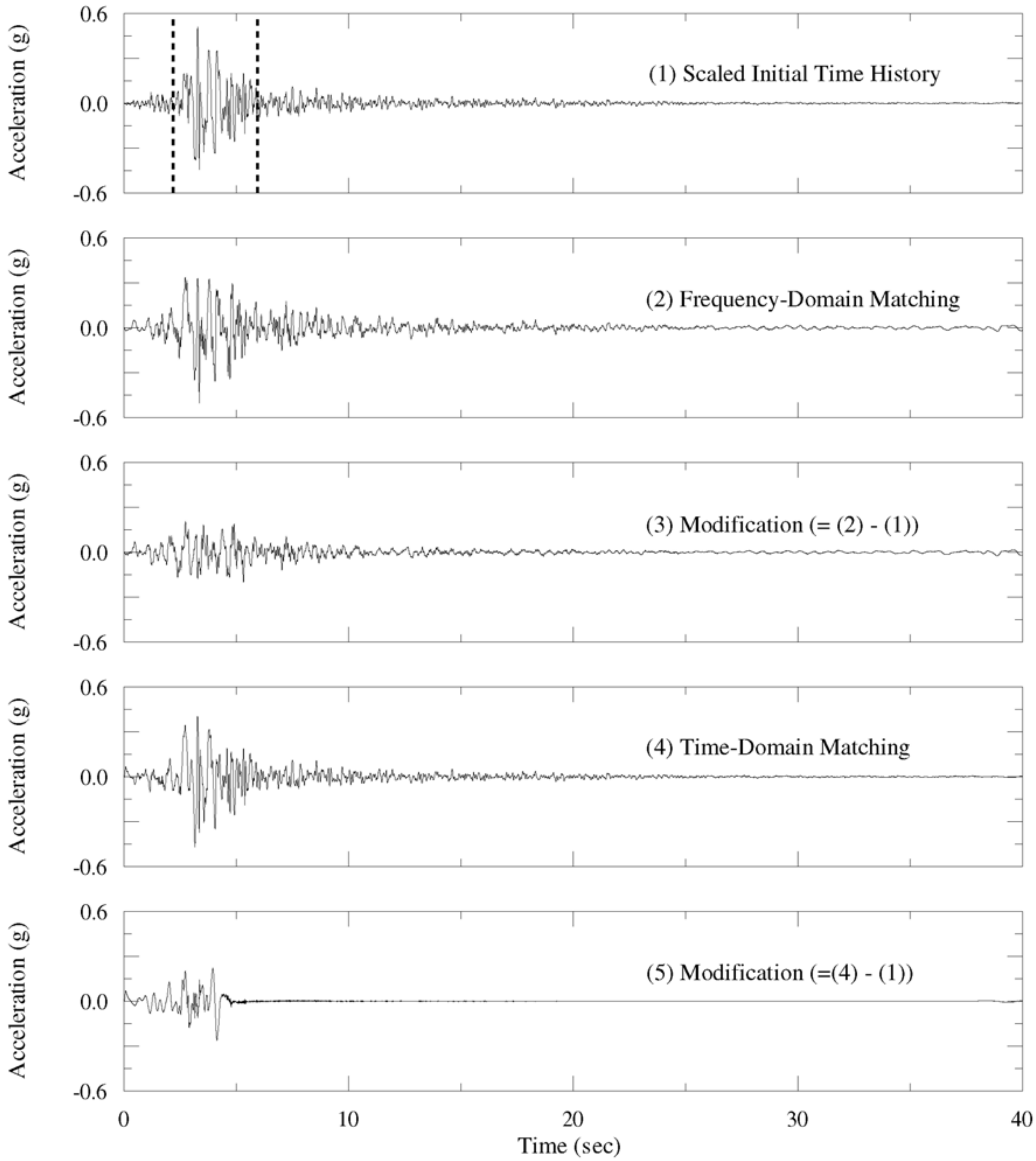
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-1. (Sheet 2 of 3)



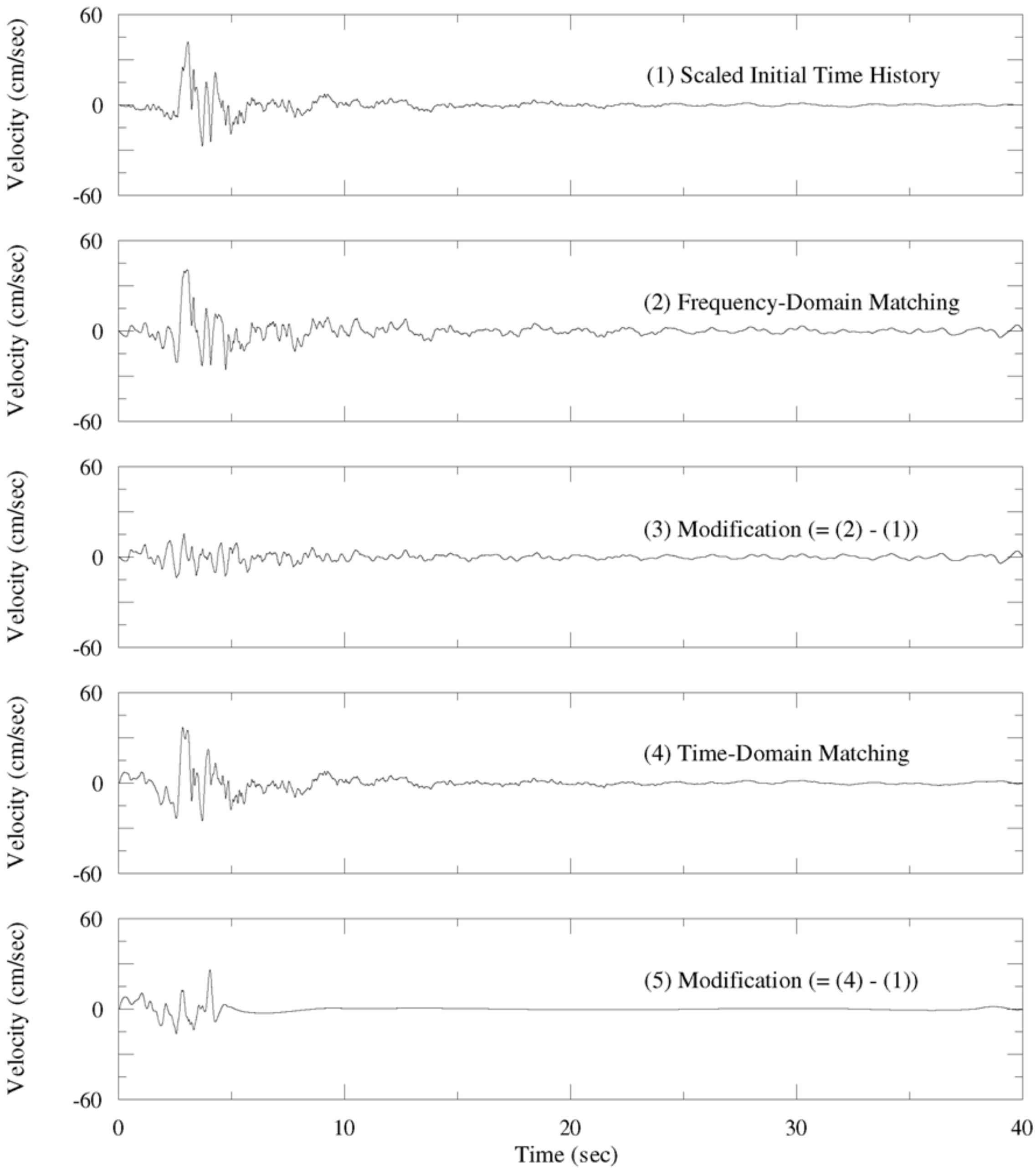
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-1. (Sheet 3 of 3)



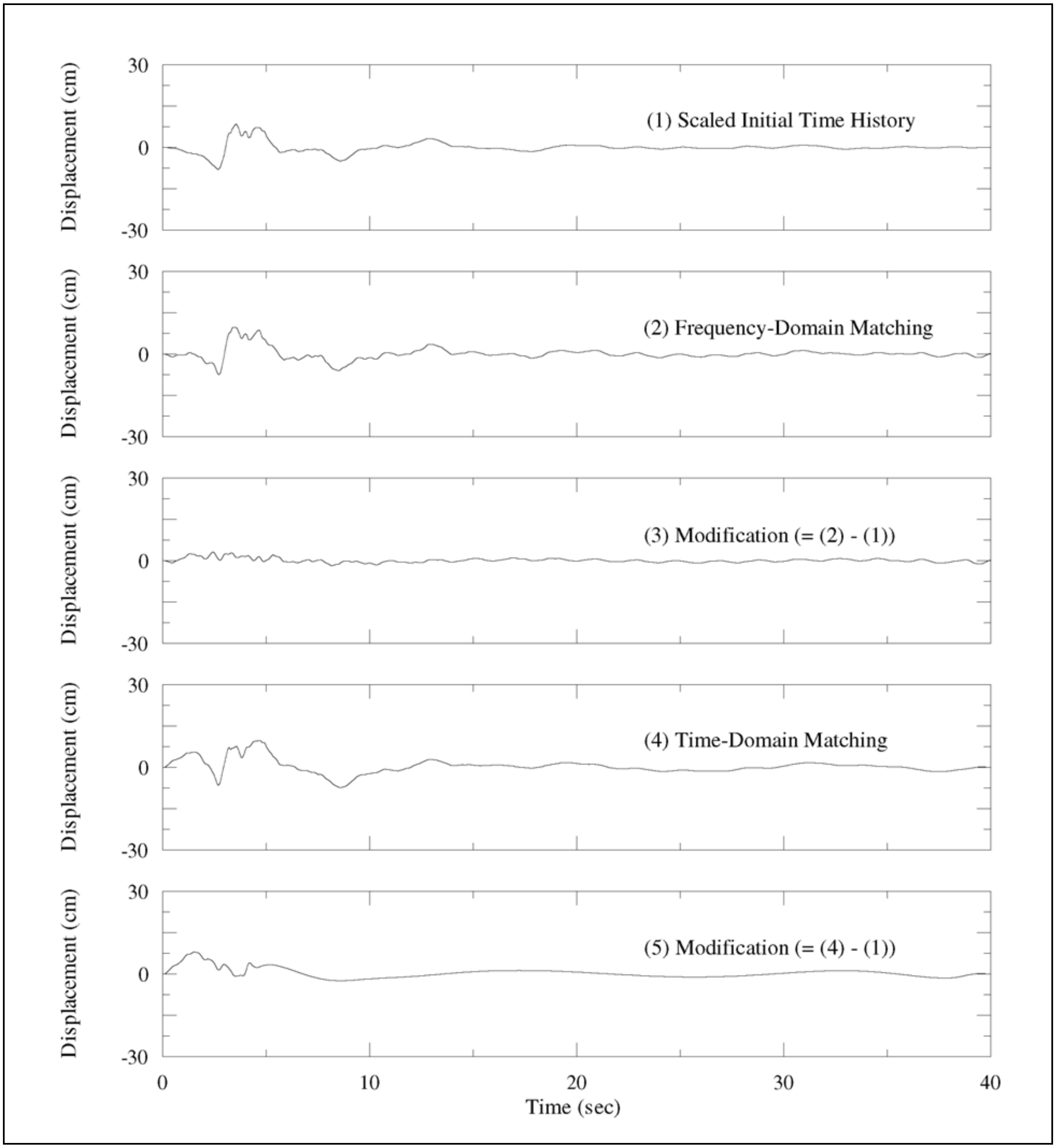
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-2. Comparison of time-histories for Gilroy recording (component 067E) 1989 Loma Prieta earthquake (Sheet 1 of 3)



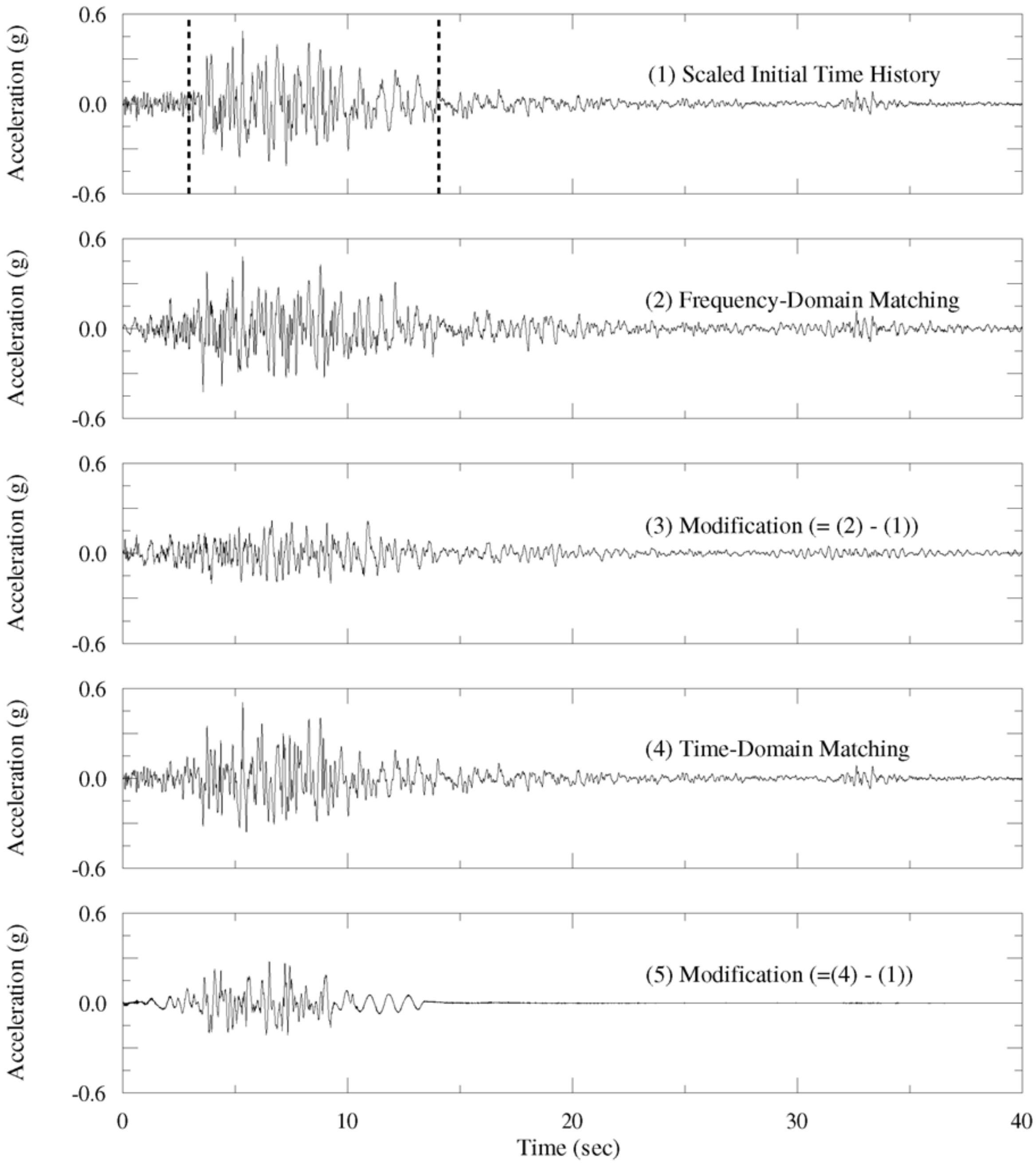
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-2. (Sheet 2 of 3)



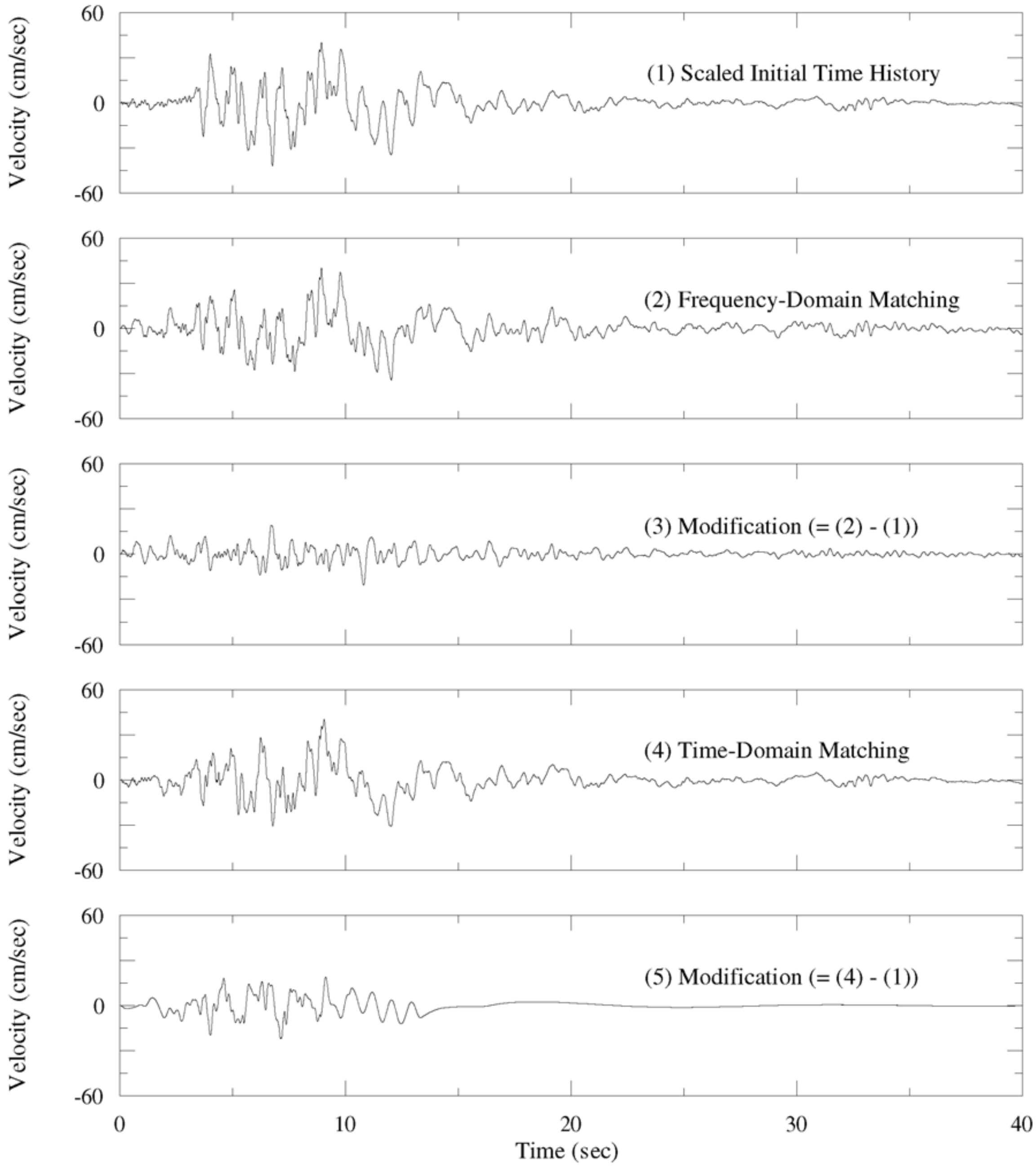
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-2. (Sheet 3 of 3)



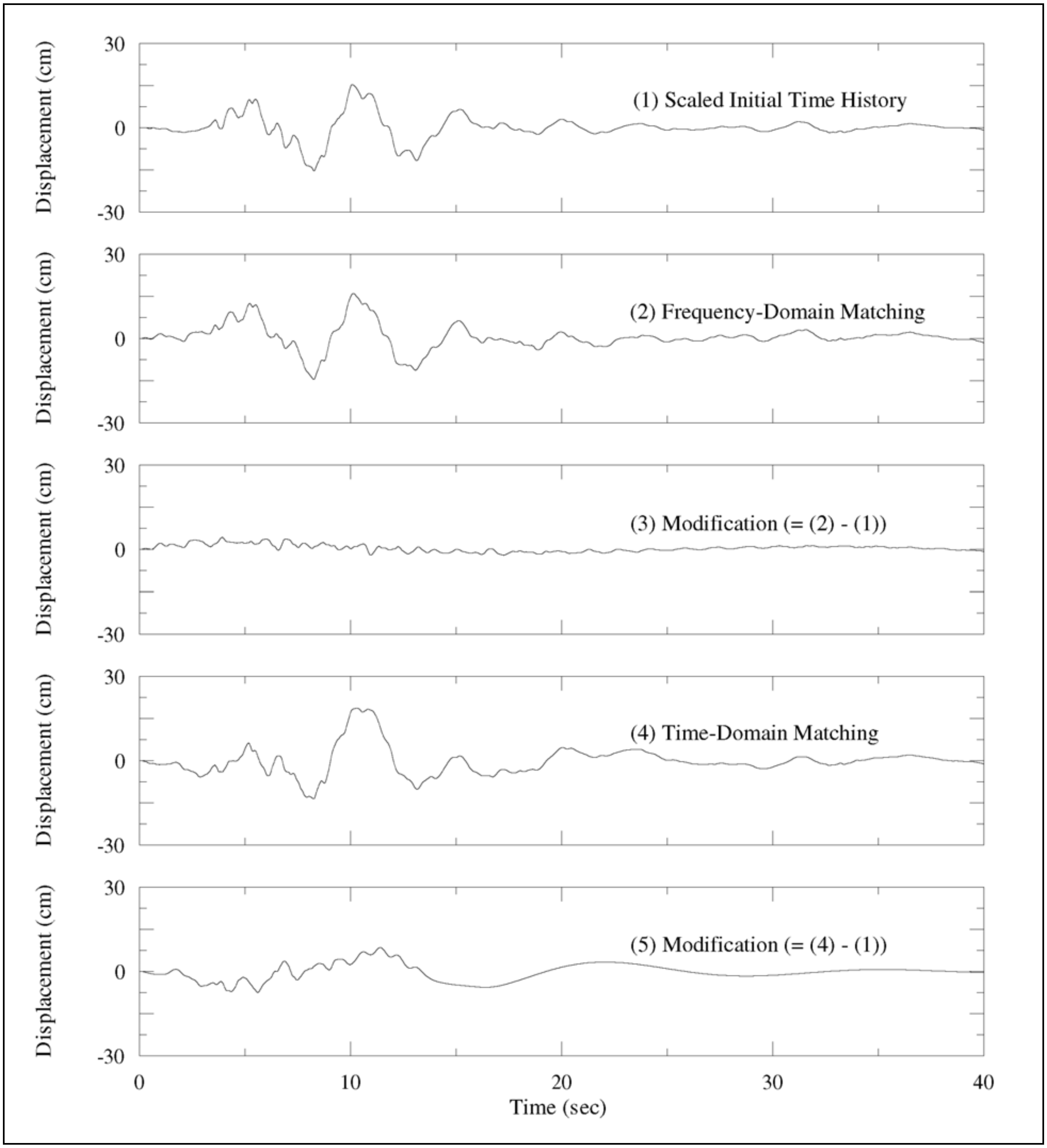
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-3. Comparison of time-histories for Griffith Park recording (component 270E), 1971 San Fernando earthquake (Sheet 1 of 3)



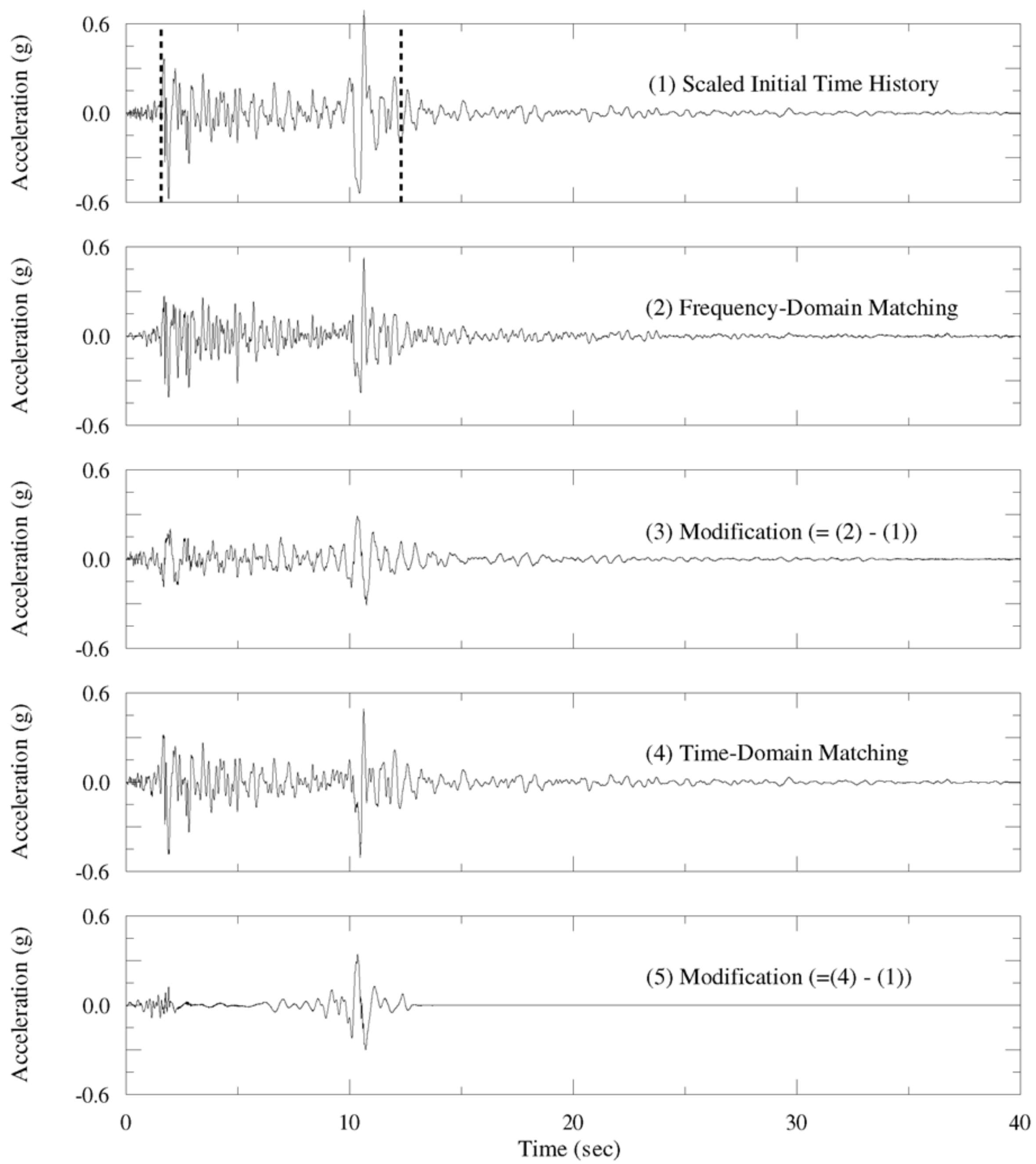
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-3. (Sheet 2 of 3)



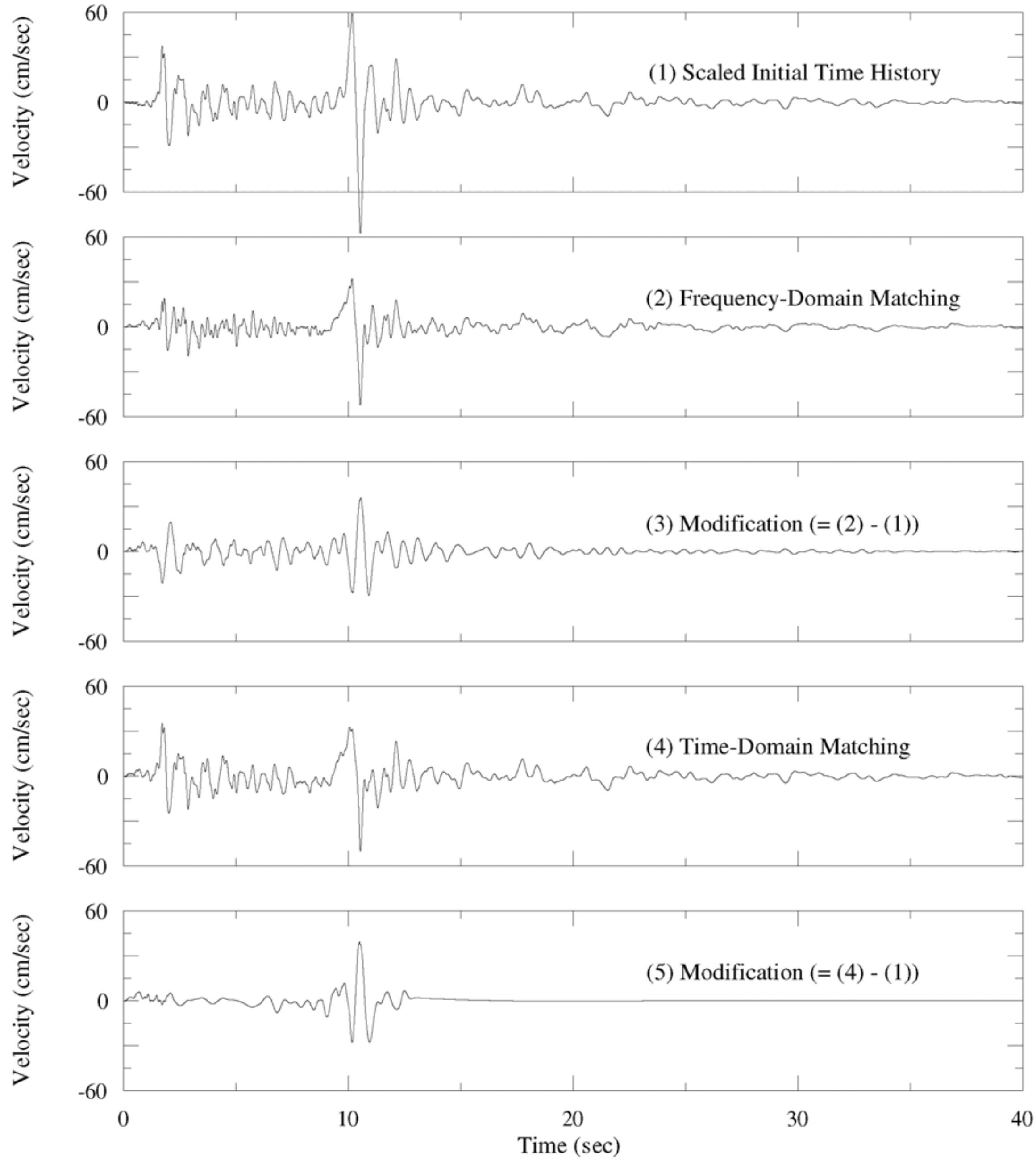
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-3. (Sheet 3 of 3)



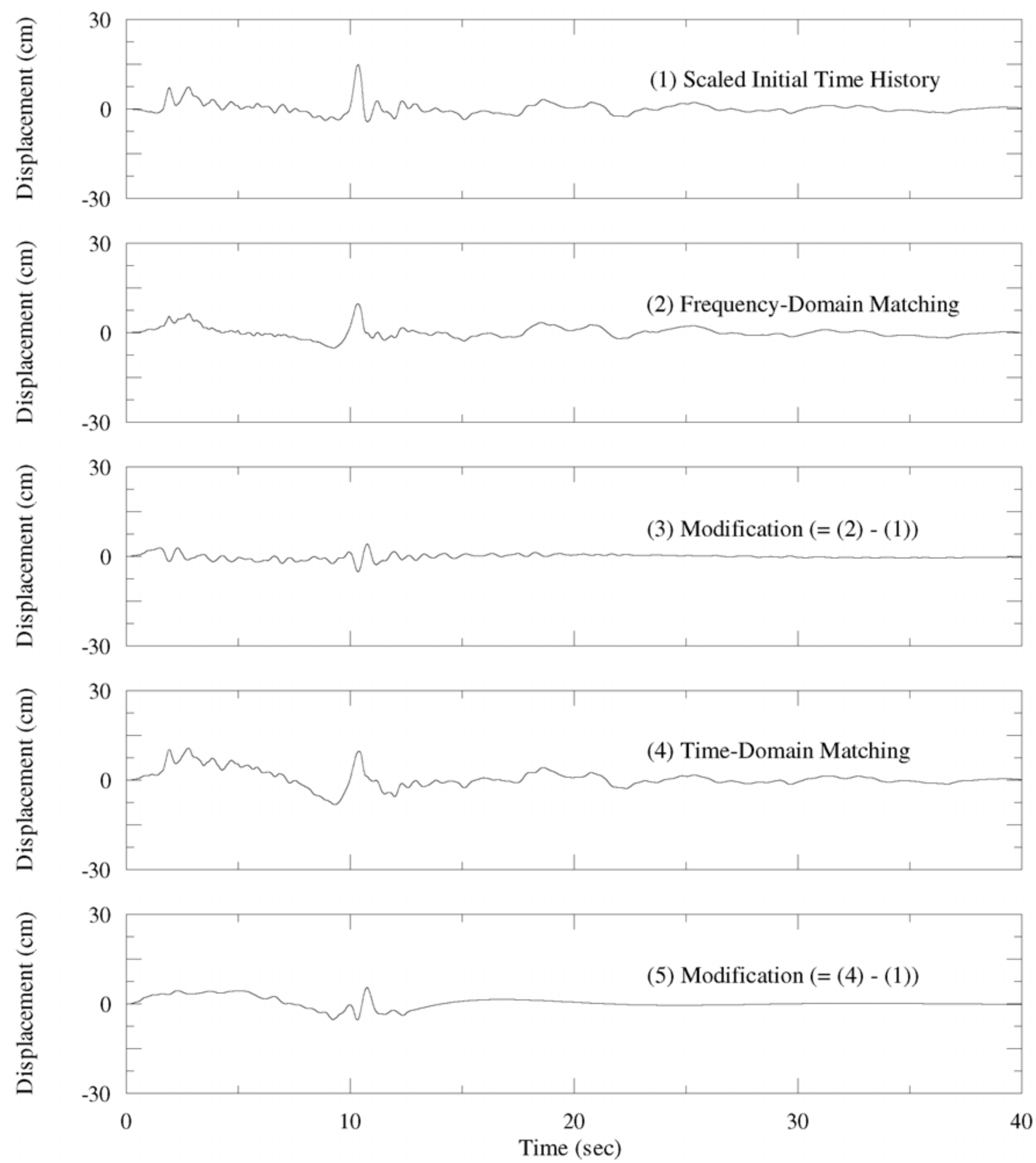
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-4. Comparison of time-histories for Halls Valley recording (component 240E), 1984 Morgan Hill earthquake (Sheet 1 of 3)



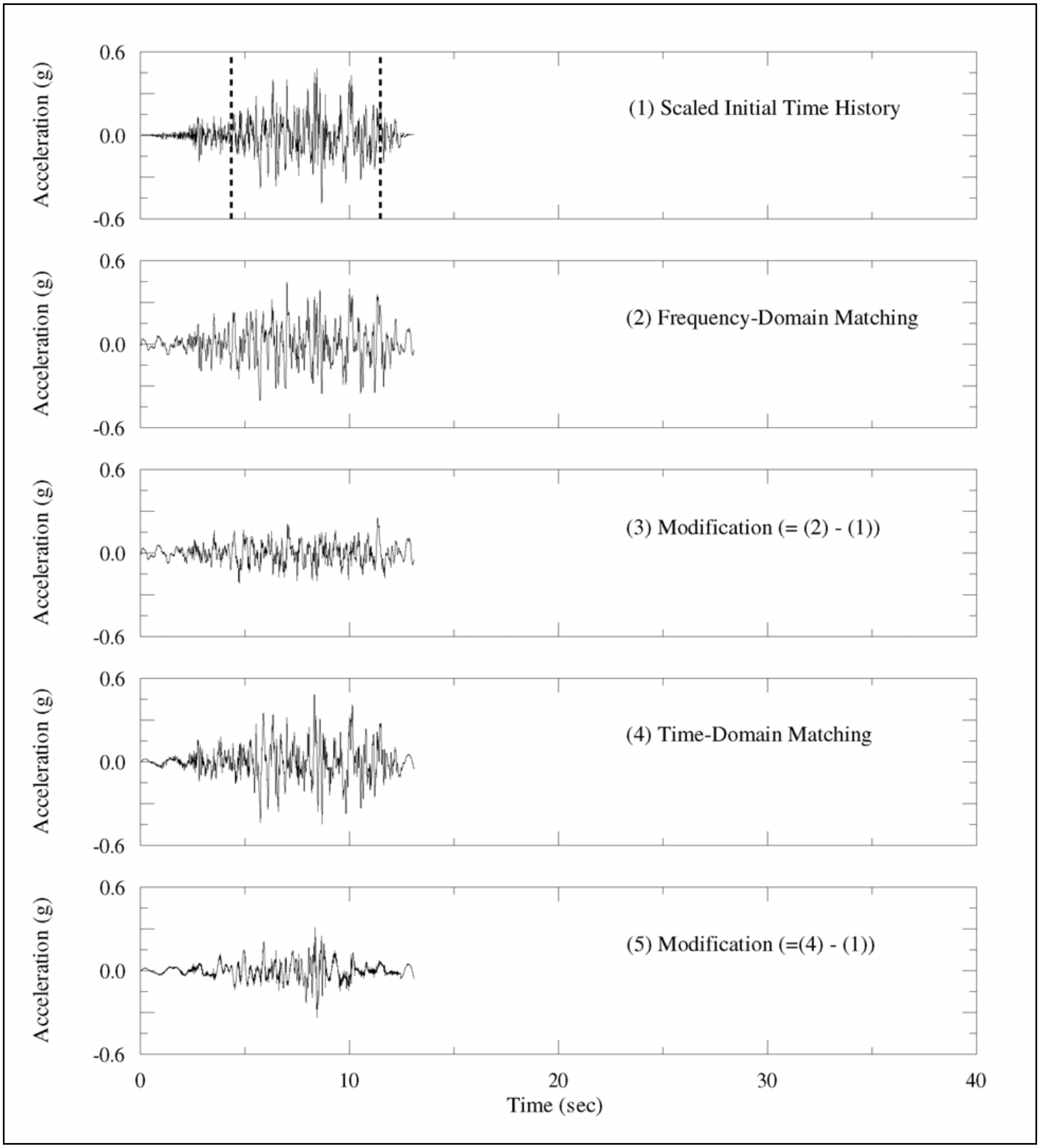
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-4. (Sheet 2 of 3)



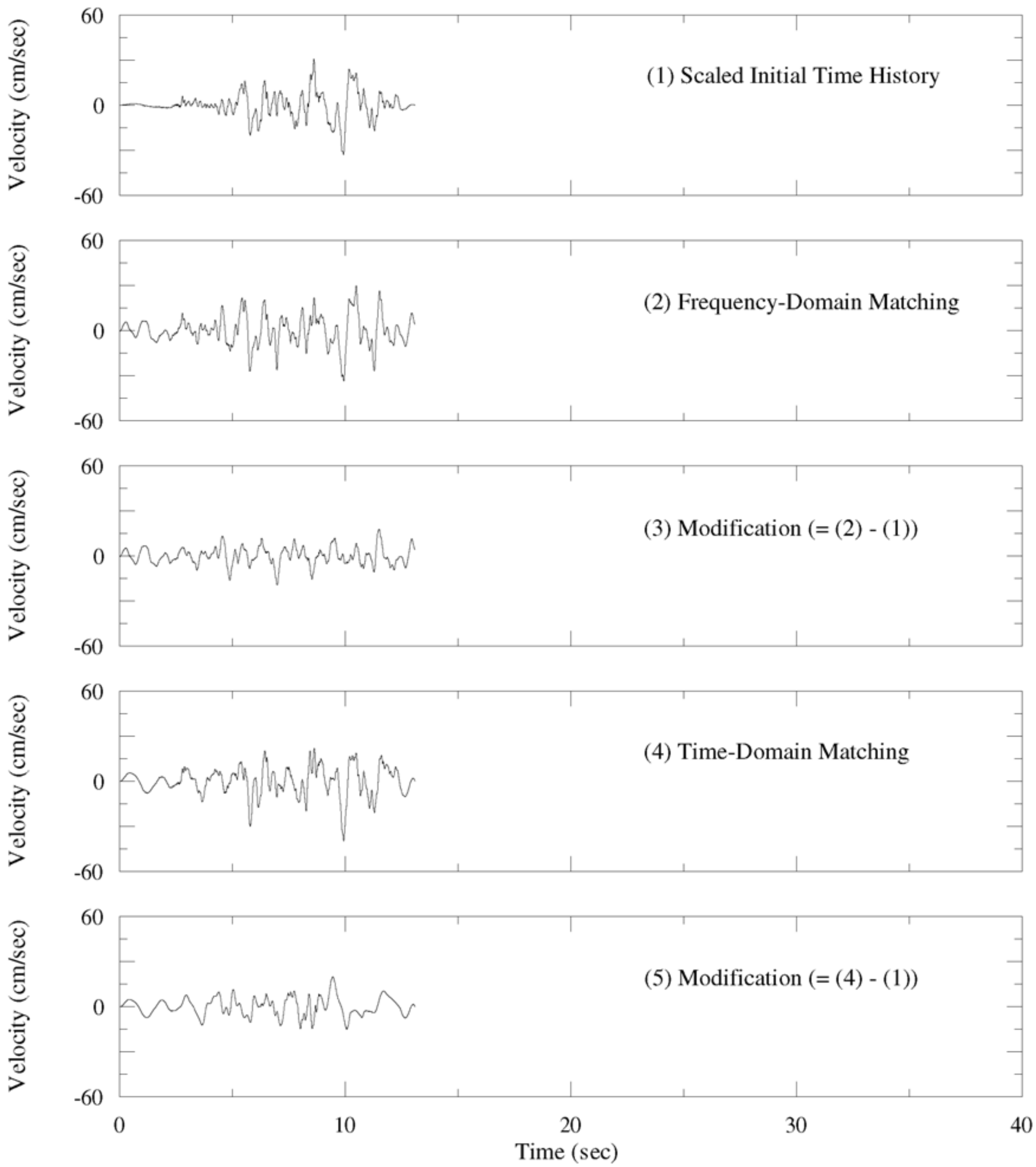
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-4. (Sheet 3 of 3)



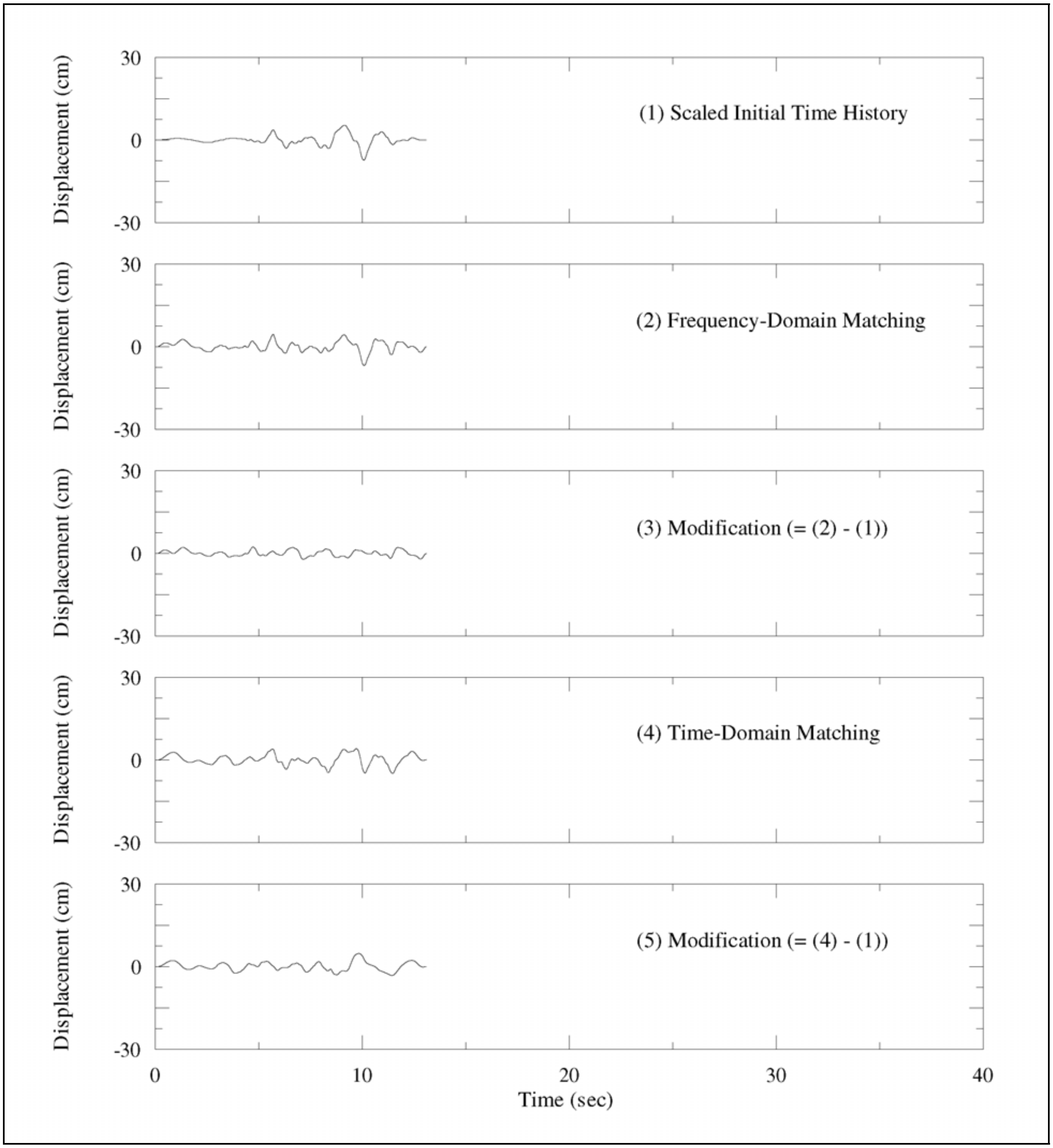
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-5. Comparison of time-histories for Gazli recording (component 090E), 1976 Gazli earthquake (Sheet 1 of 3)



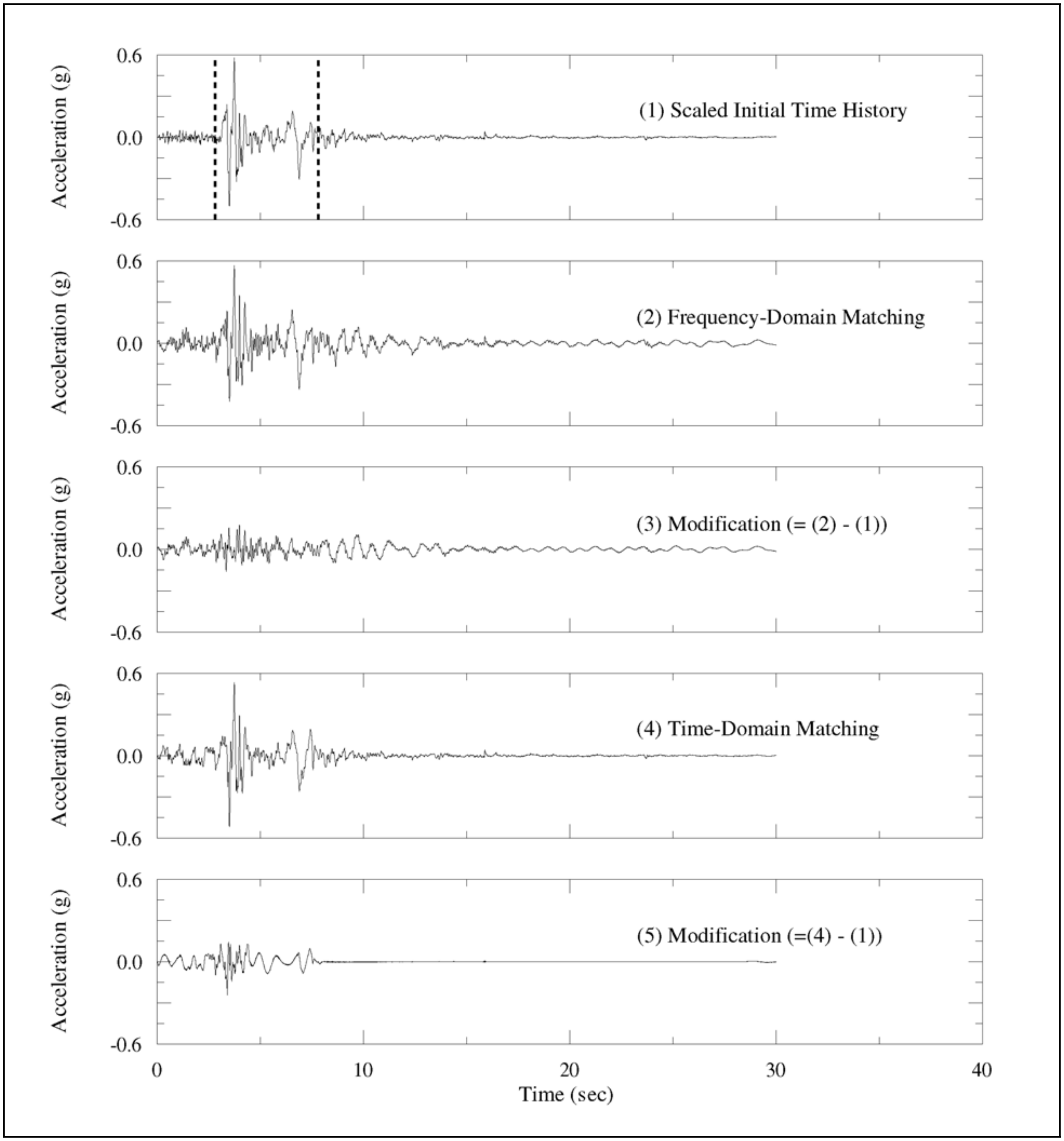
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-5. (Sheet 2 of 3)



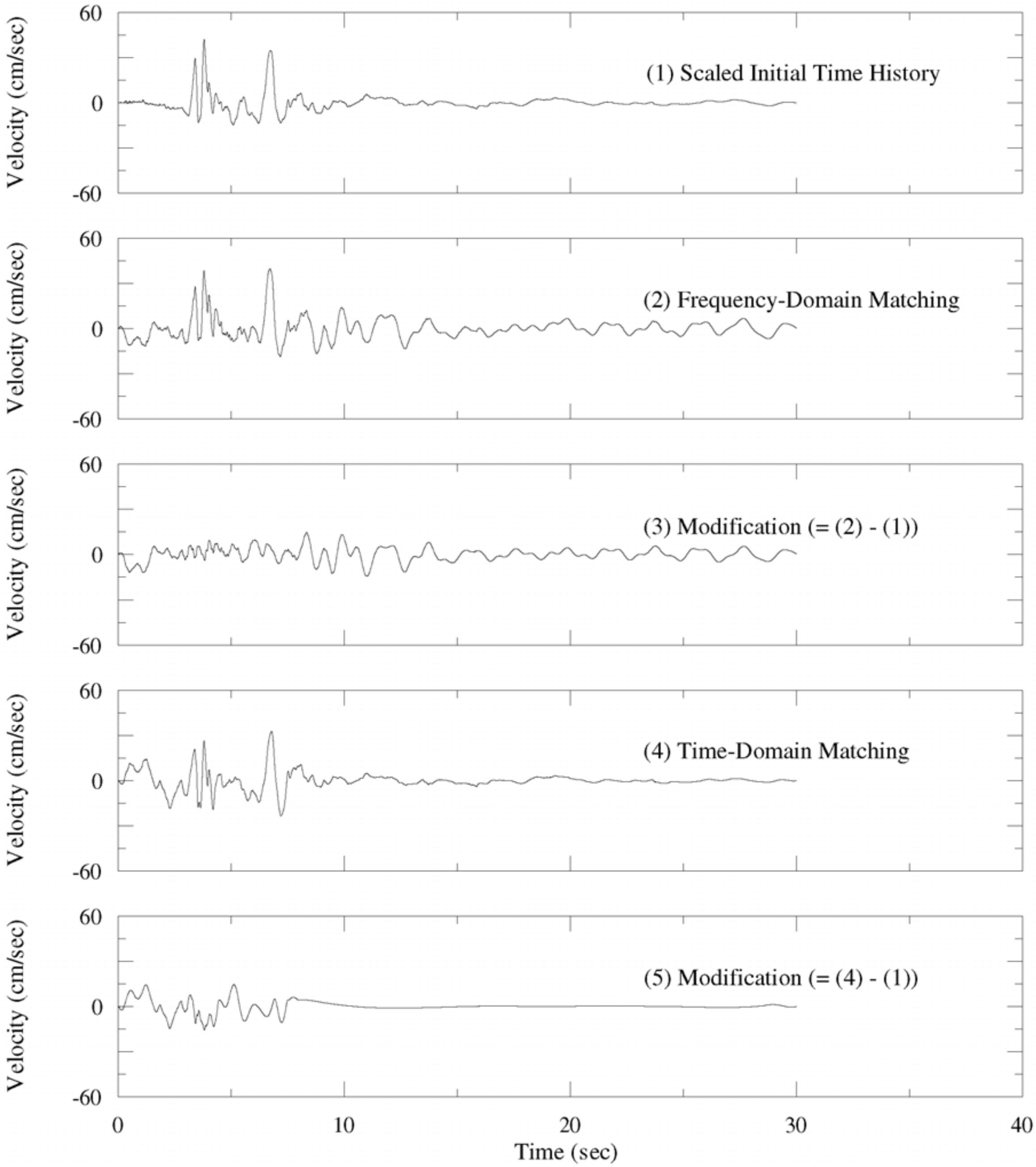
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-5. (Sheet 3 of 3)



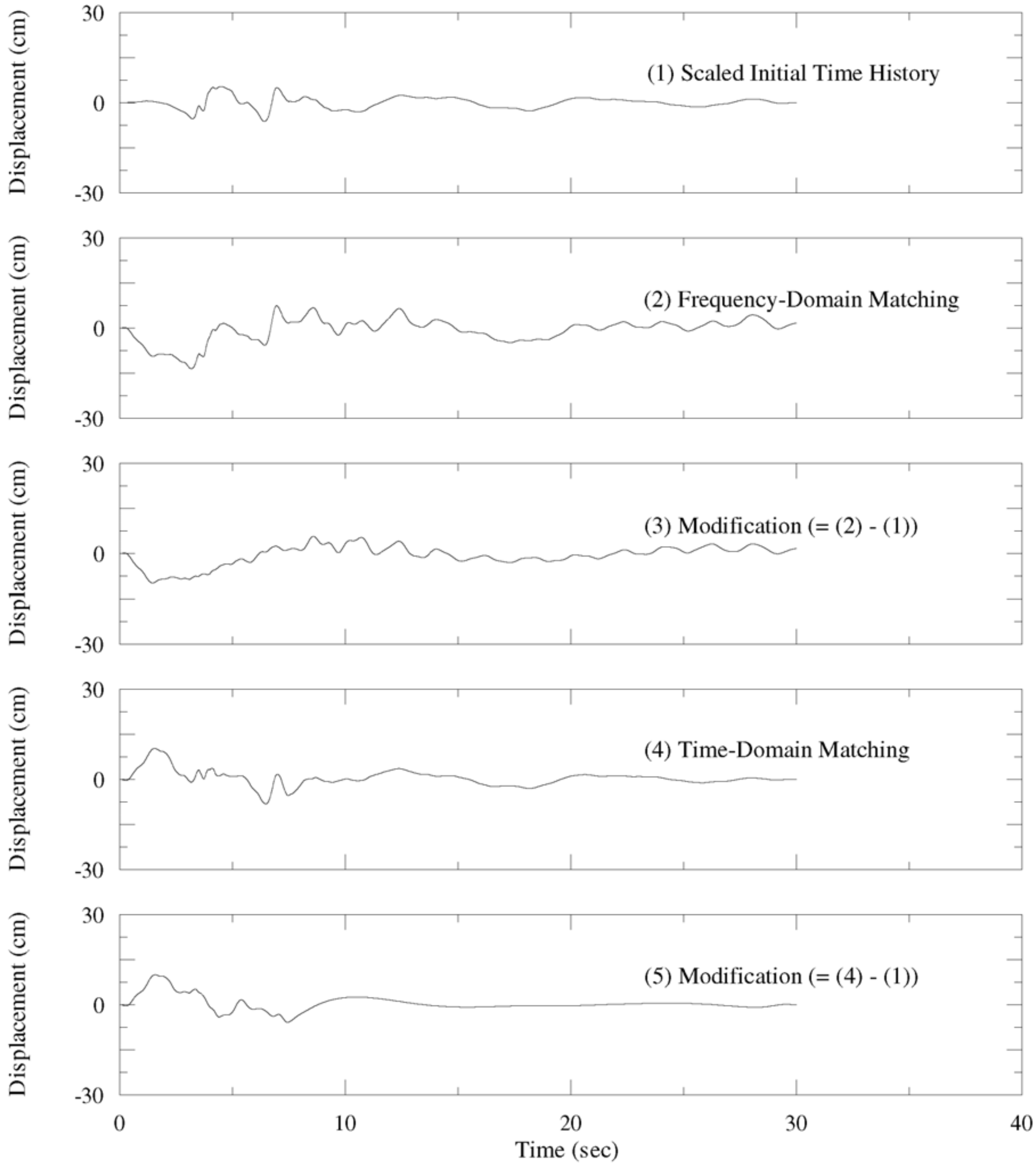
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-6. Comparison of time-histories for Pacoima Dam recording (component 265E), 1994 Northridge earthquake (Sheet 1 of 3)



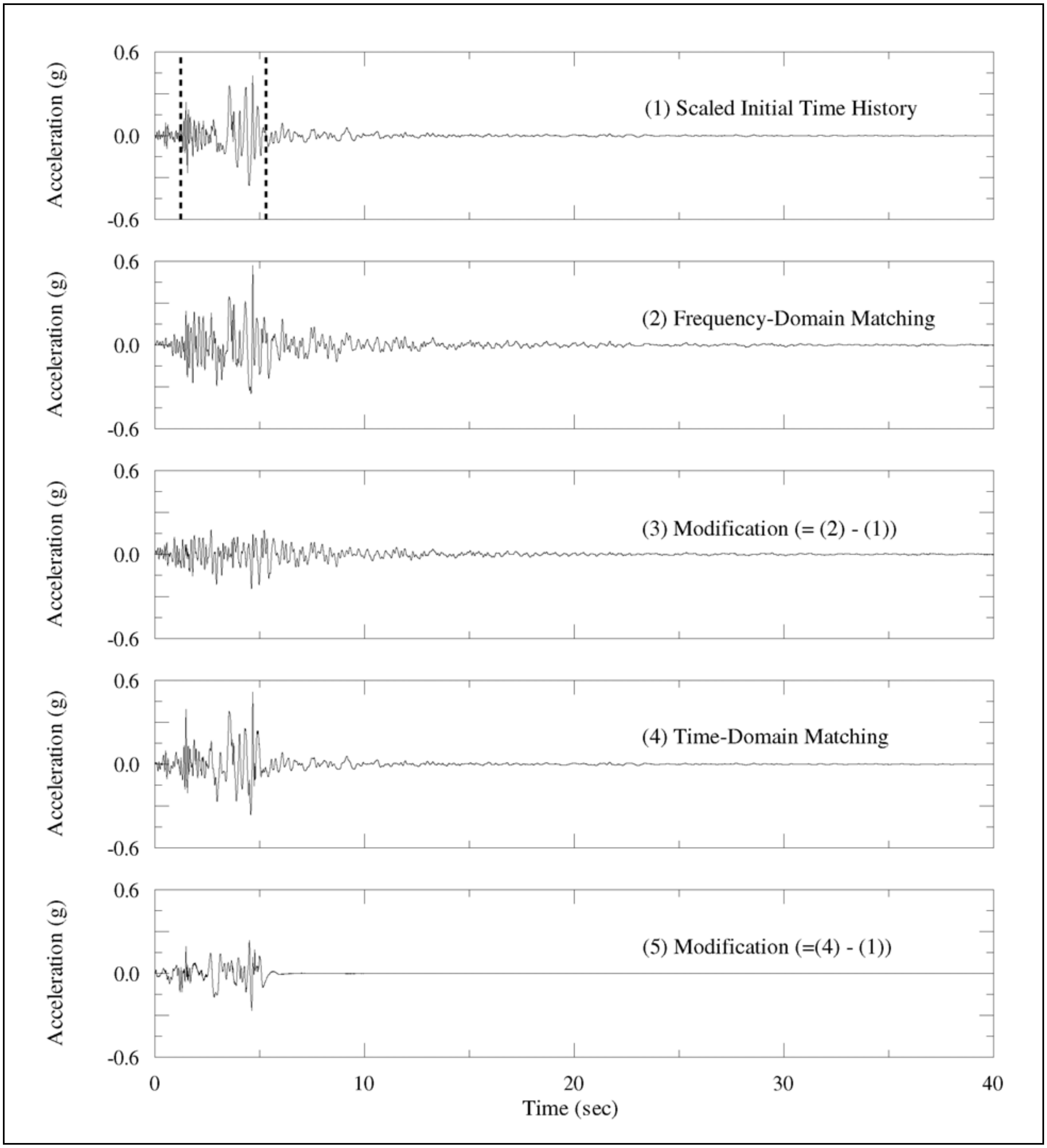
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-6. (Sheet 2 of 3)



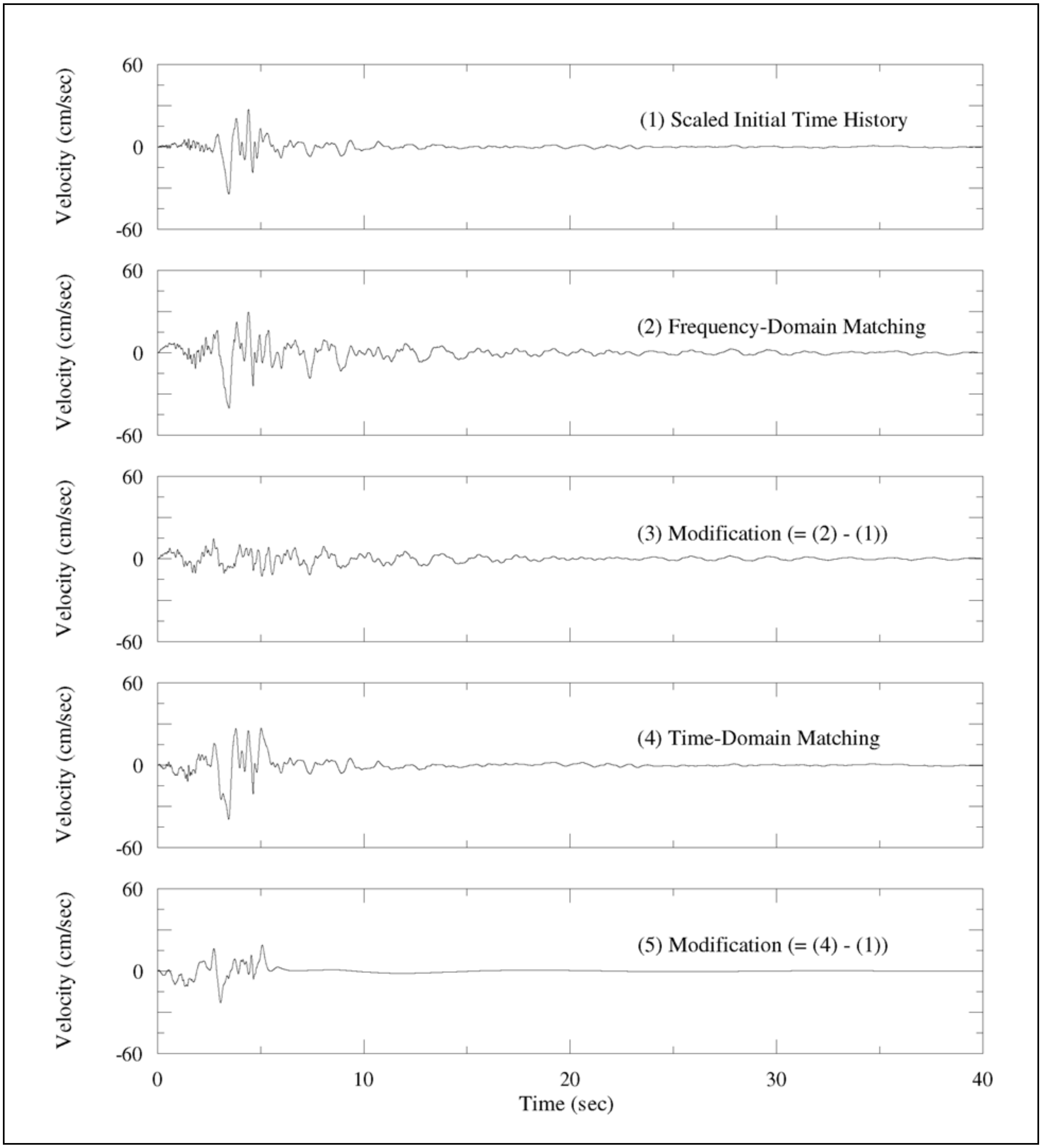
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-6. (Sheet 3 of 3)



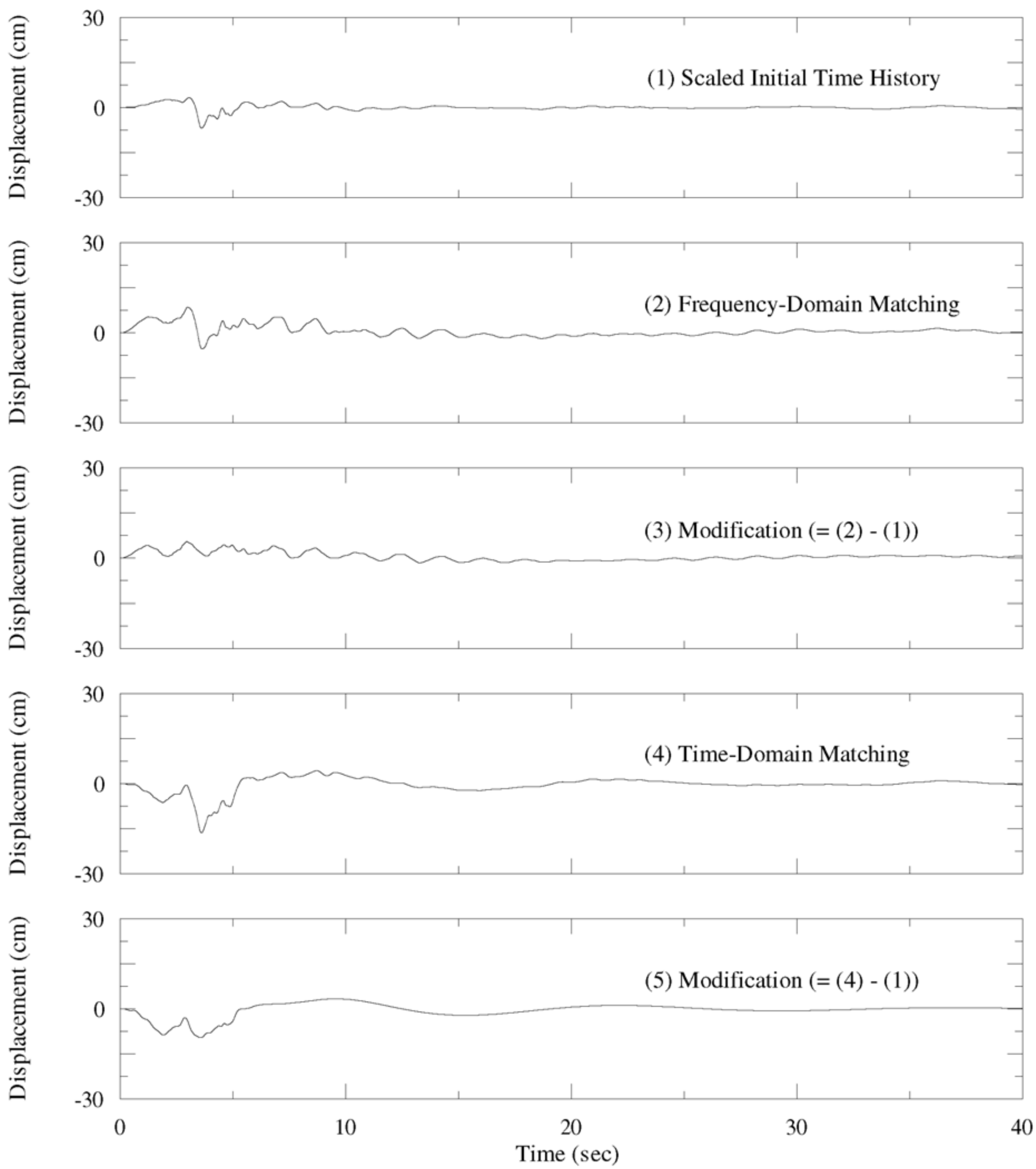
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-7. Comparison of time-histories for Coyote Lake Dam recording (component 195E), 1984 Morgan Hill earthquake (Sheet 1 of 3)



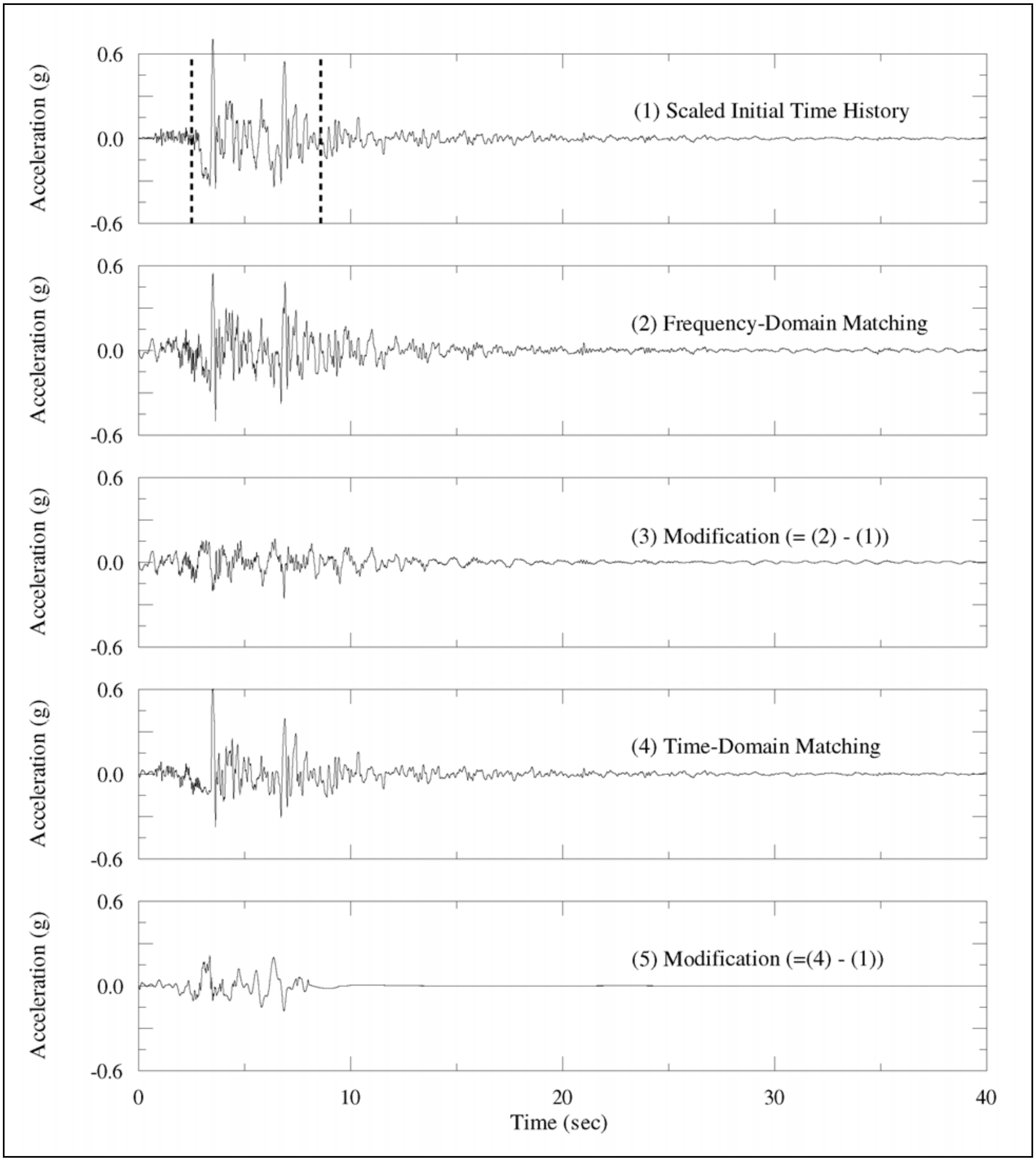
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-7. (Sheet 2 of 3)



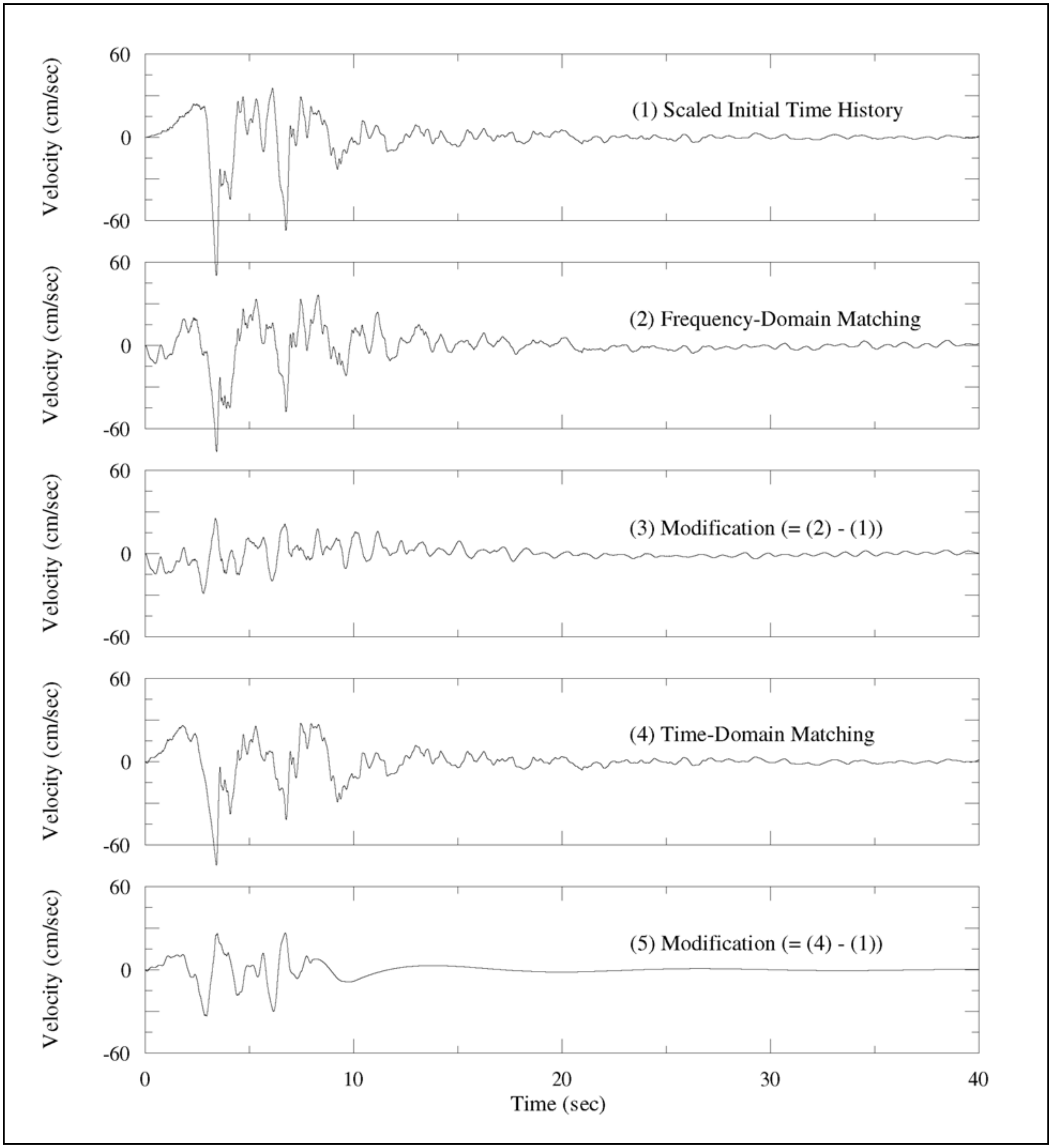
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-7. (Sheet 3 of 3)



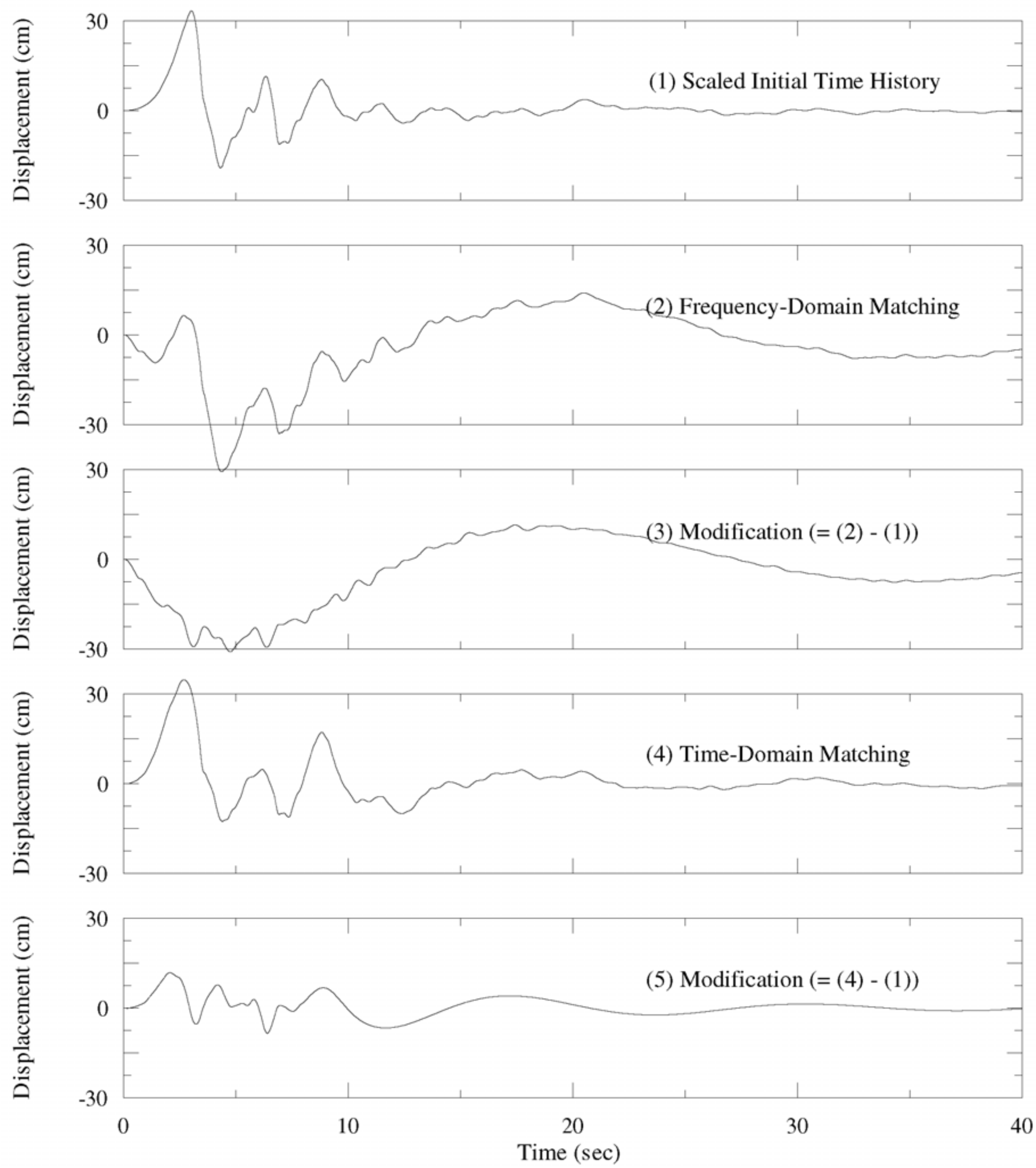
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-8. Comparison of time-histories for SCSE recording (component 018E), 1994 Northridge earthquake (Sheet 1 of 3)



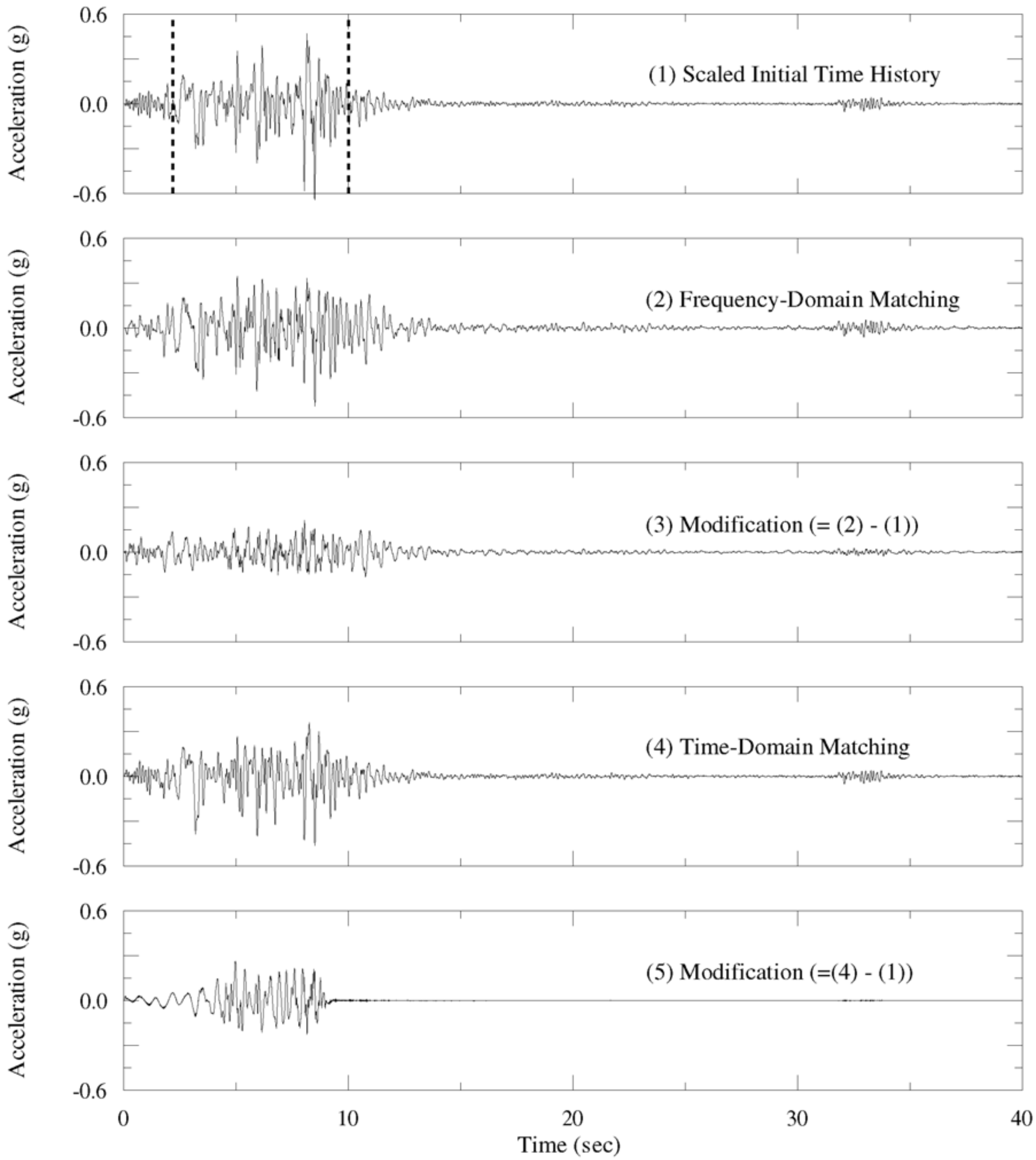
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-8. (Sheet 2 of 3)



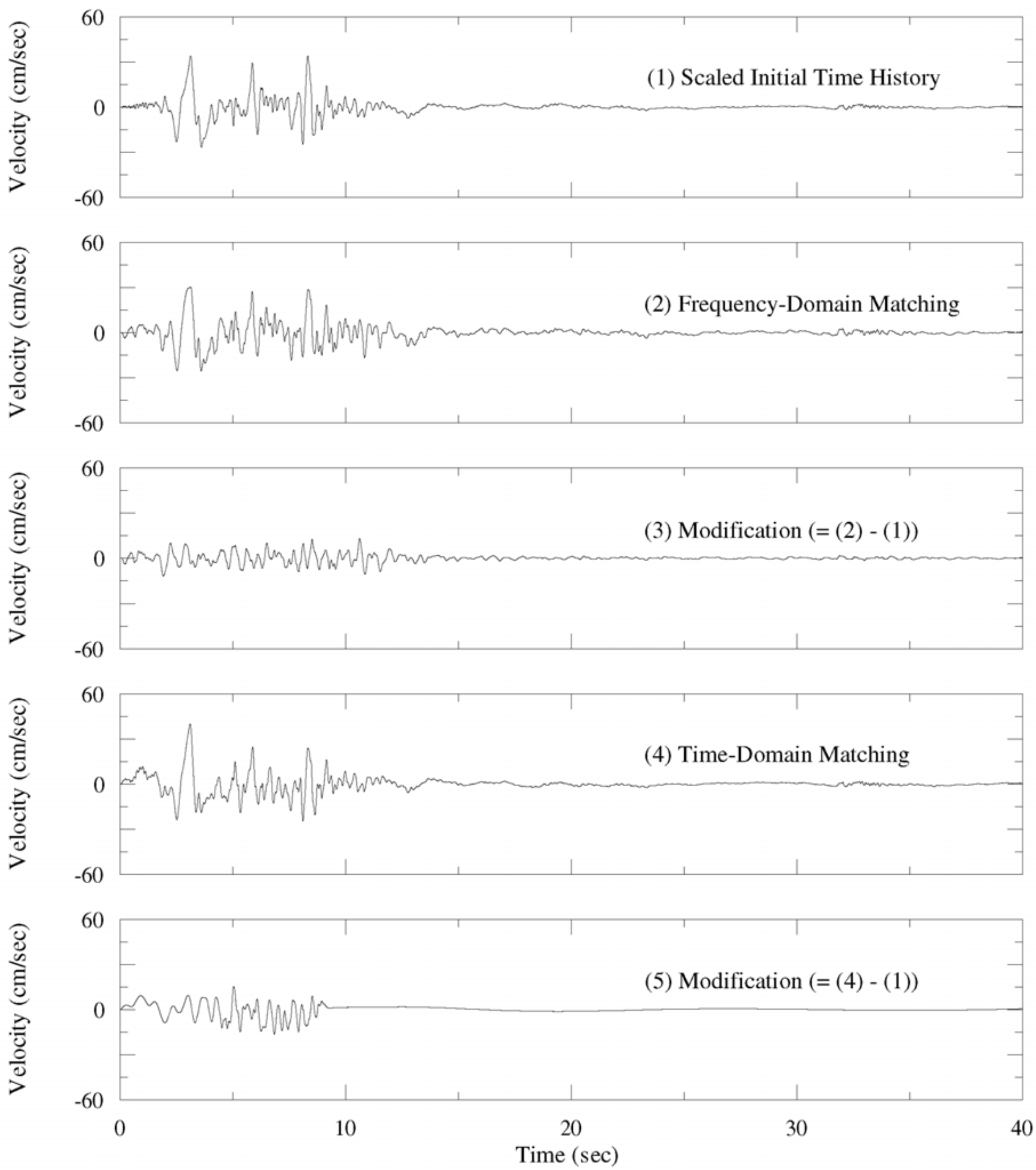
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-8. (Sheet 3 of 3)



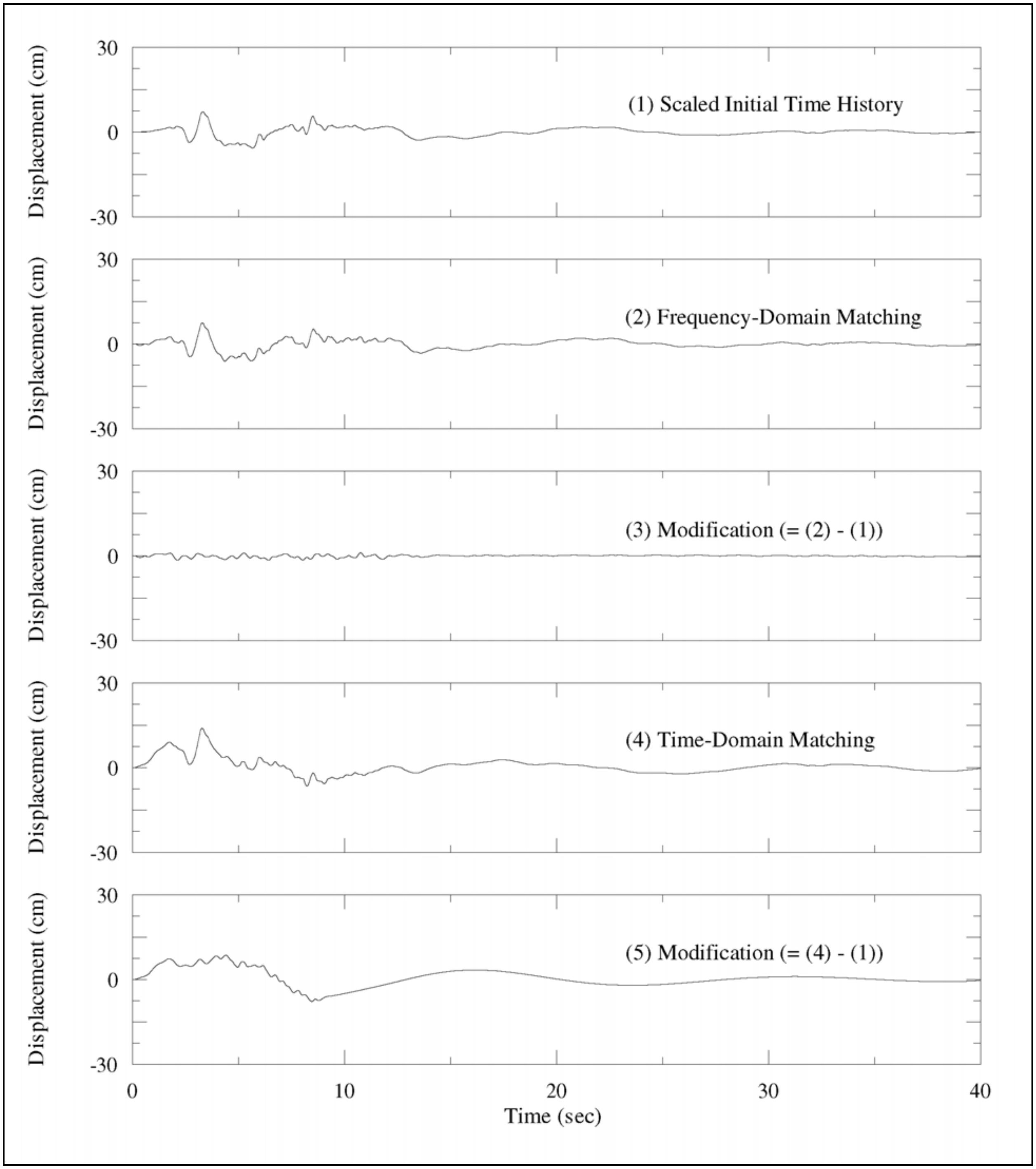
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-9. Comparison of time-histories for Pacoima Dam recording (component 254E), 1971 San Fernando earthquake (Sheet 1 of 3)



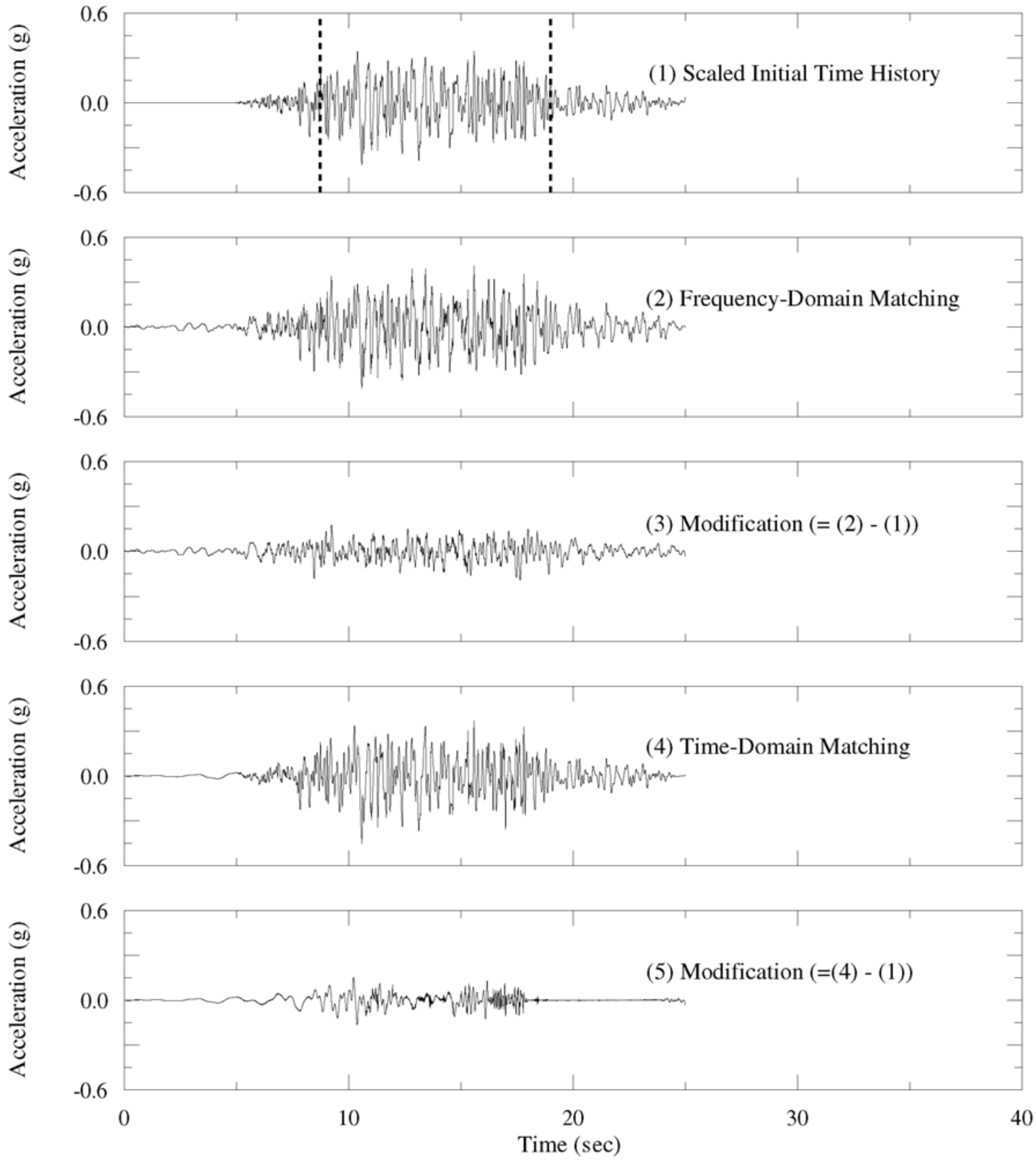
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-9. (Sheet 2 of 3)



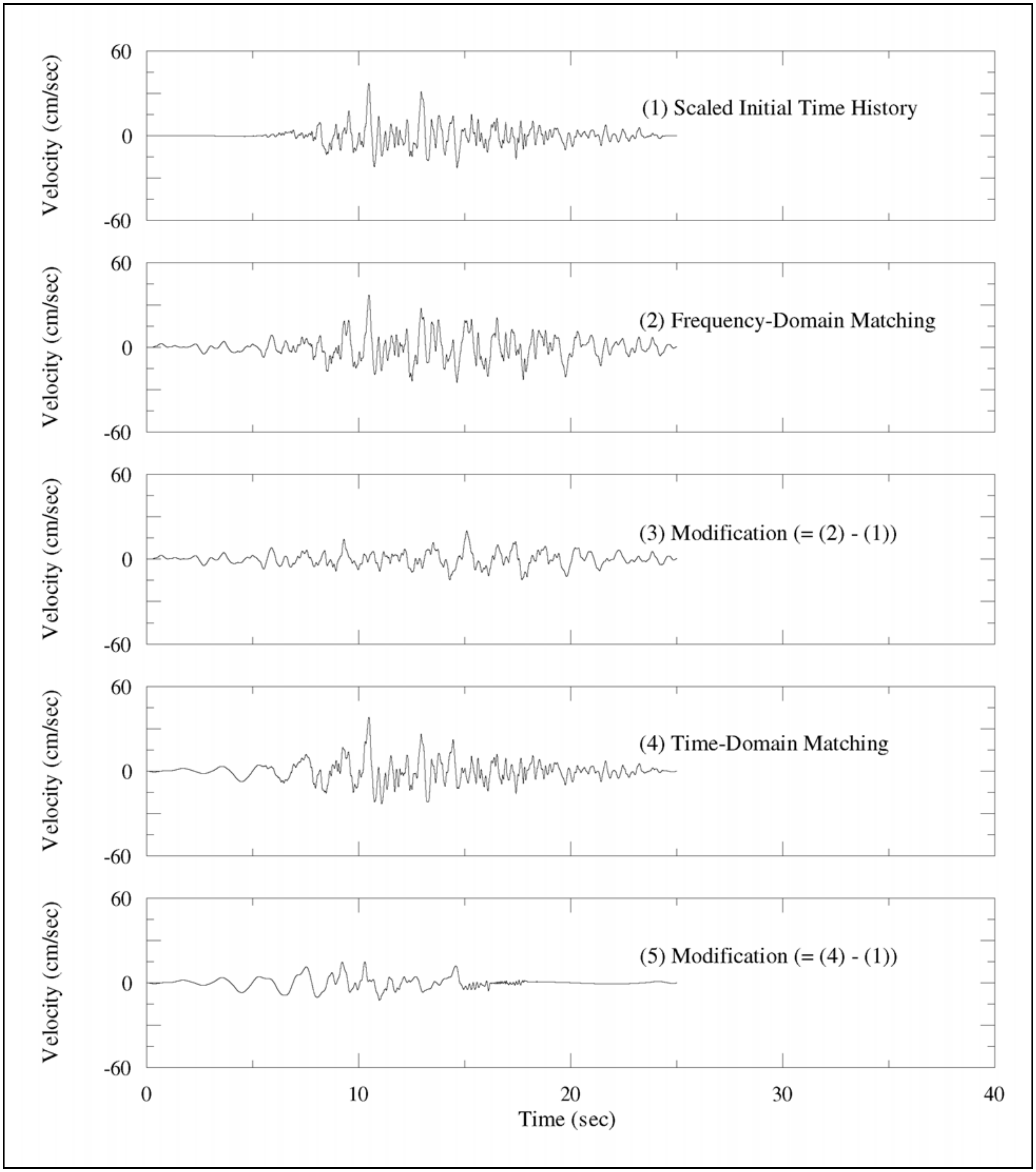
c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-9. (Sheet 3 of 3)



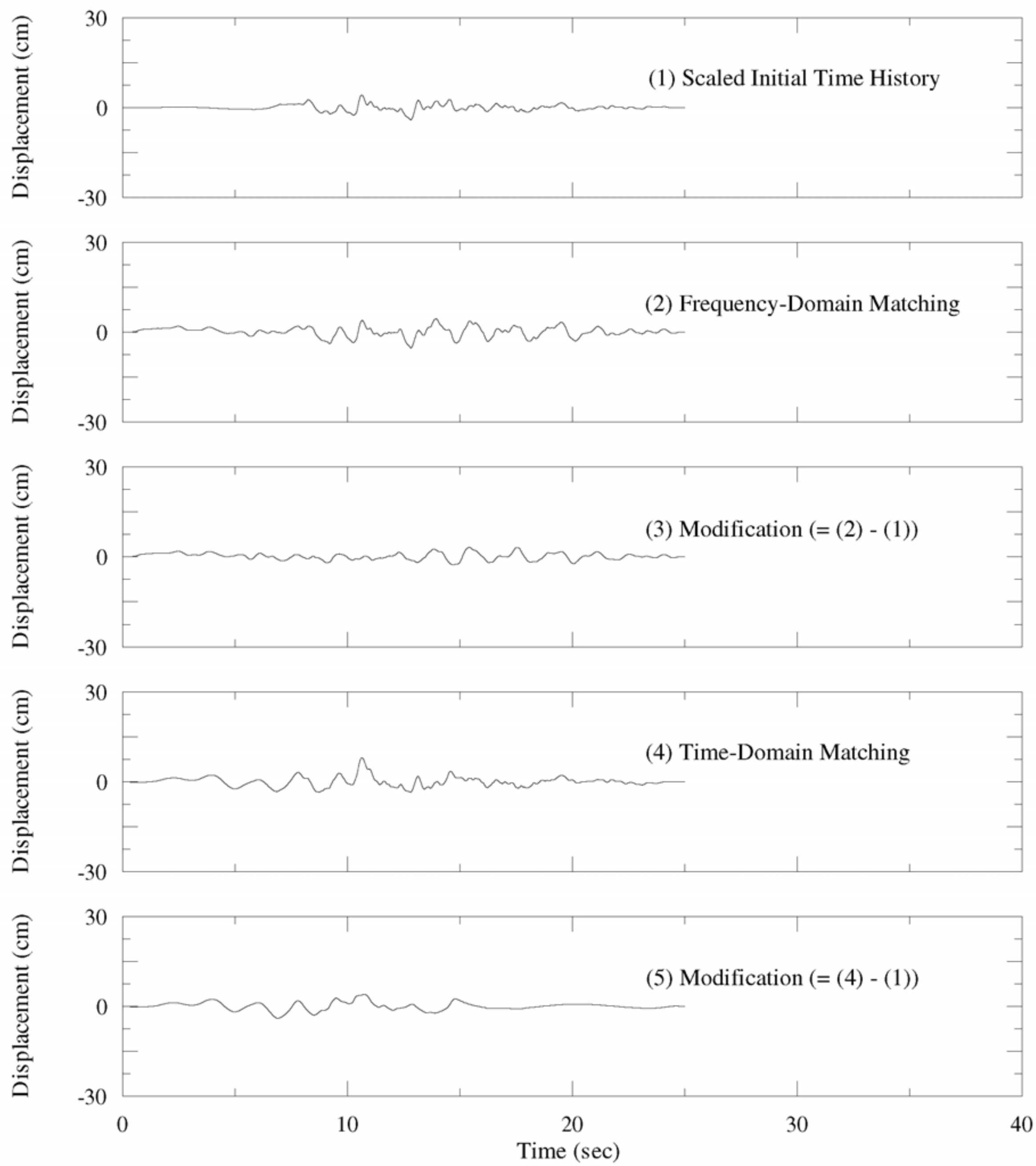
a. Comparison of acceleration time-histories: (1) the scaled initial recorded time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method. Dashed lines indicate the time window used in the calculation of power spectral density function.

Figure C-10. Comparison of time-histories for UCSC BRAN recording (component 090E), 1989 Loma Prieta earthquake (Sheet 1 of 3)



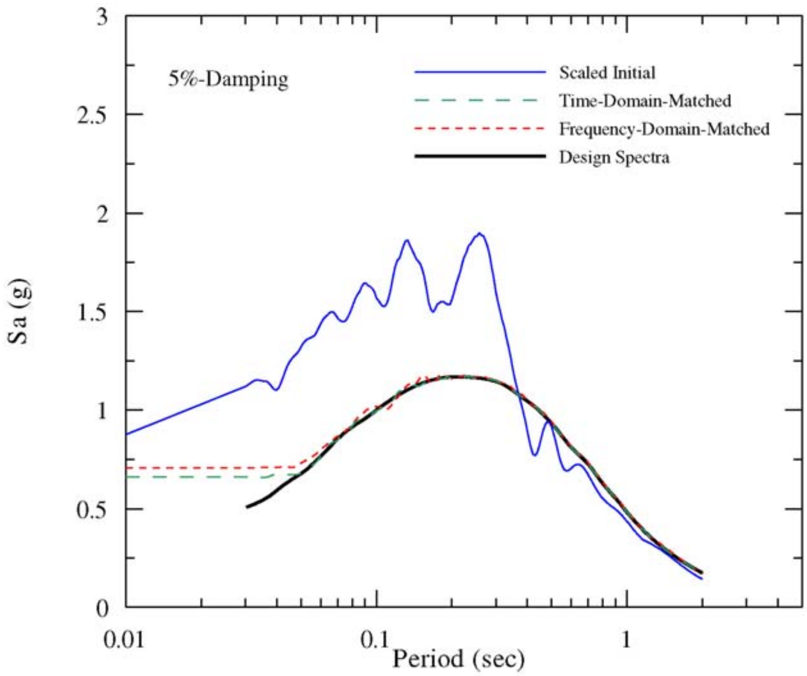
b. Comparison of velocity time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-10. (Sheet 2 of 3)

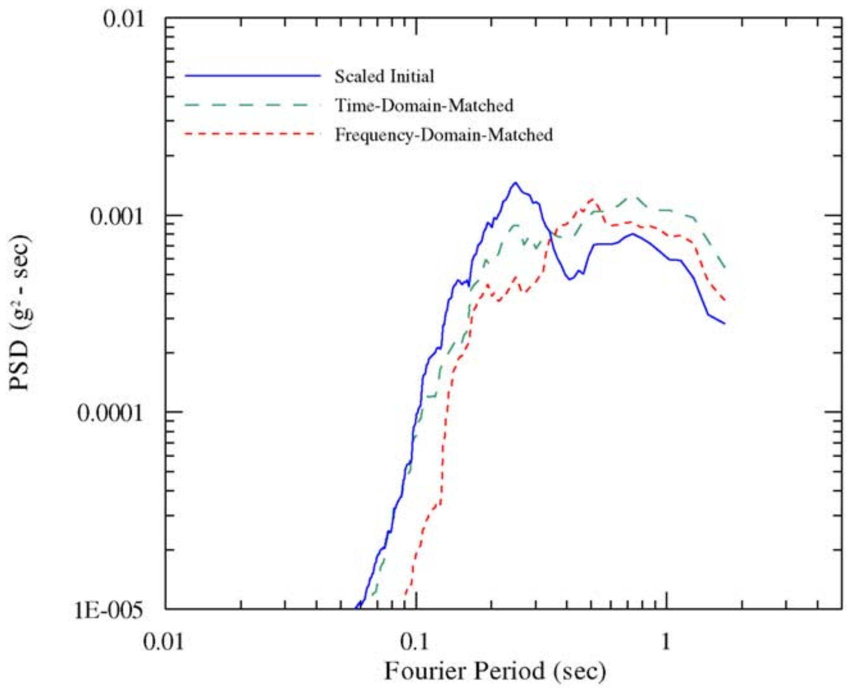


c. Comparison of displacement time-histories: (1) the scaled initial time-history; (2) frequency-domain-matched time-history; (3) modification to the time-history made by the frequency-domain method; (4) time-domain-matched time-history; (5) modification to the time-history made by time-domain method.

Figure C-10. (Sheet 3 of 3)



a. 5 percent damped response spectra



b. Power spectral density functions

Figure C-11. Comparisons of response spectra and power spectral density functions for the scaled Petrolia recording (component 000E), 1992 Cape Mendocino earthquake, and the spectrum-compatible acceleration time-histories

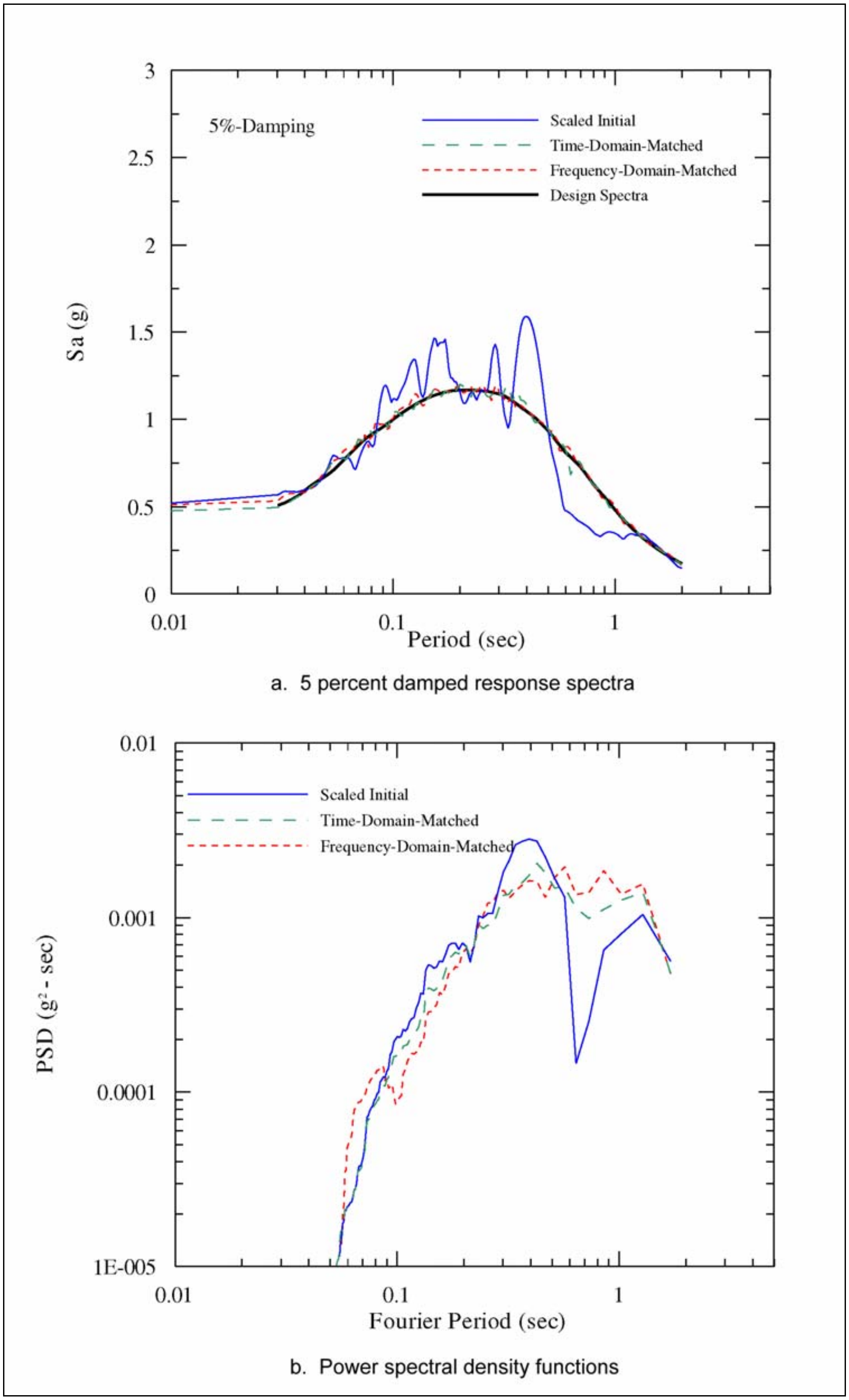
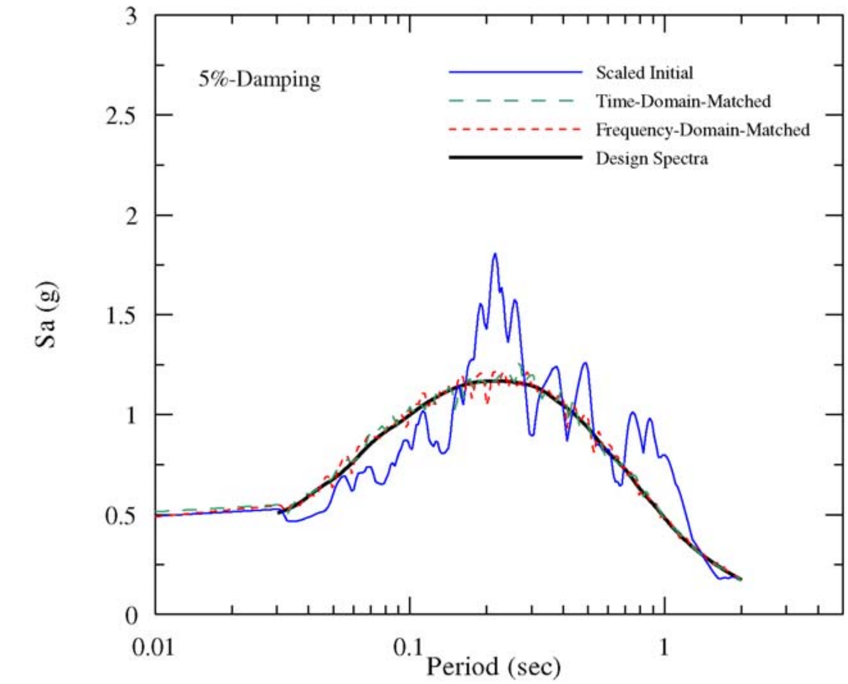
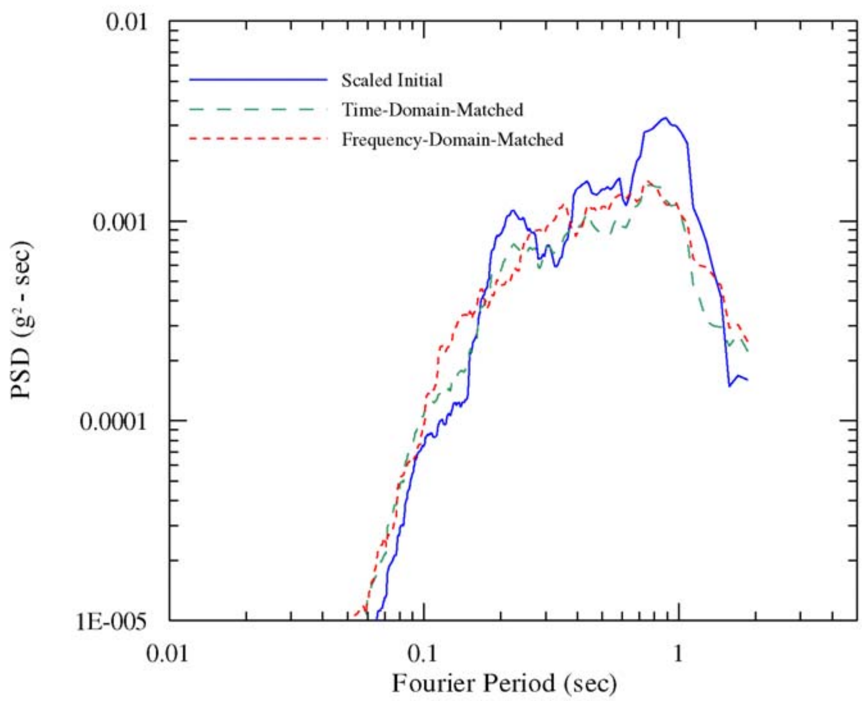


Figure C-12. Comparisons of response spectra and power spectral density functions for the scaled Gilroy recording (component 067E), 1989 Loma Prieta earthquake, and the spectrum-compatible acceleration time-histories

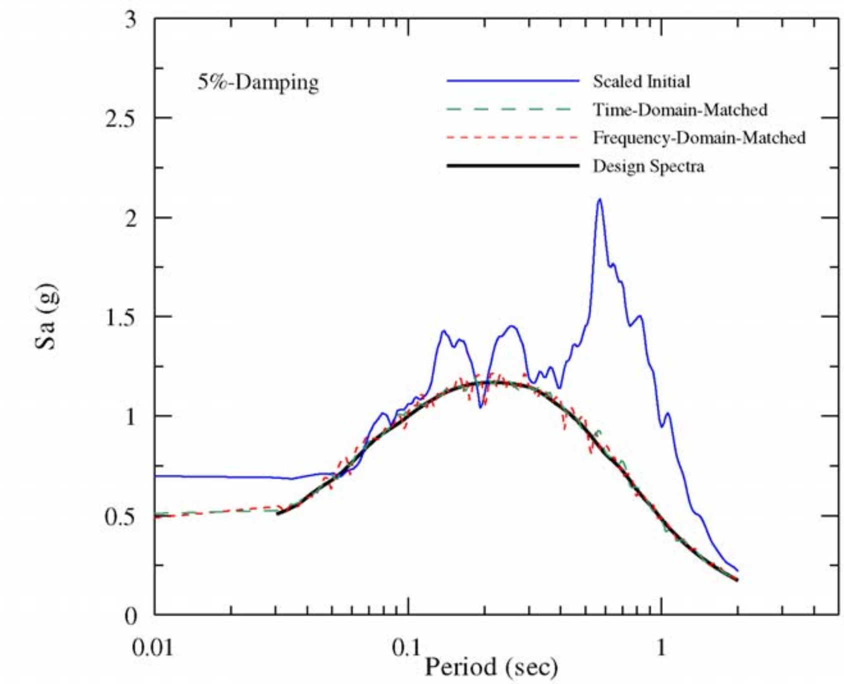


a. 5 percent damped response spectra

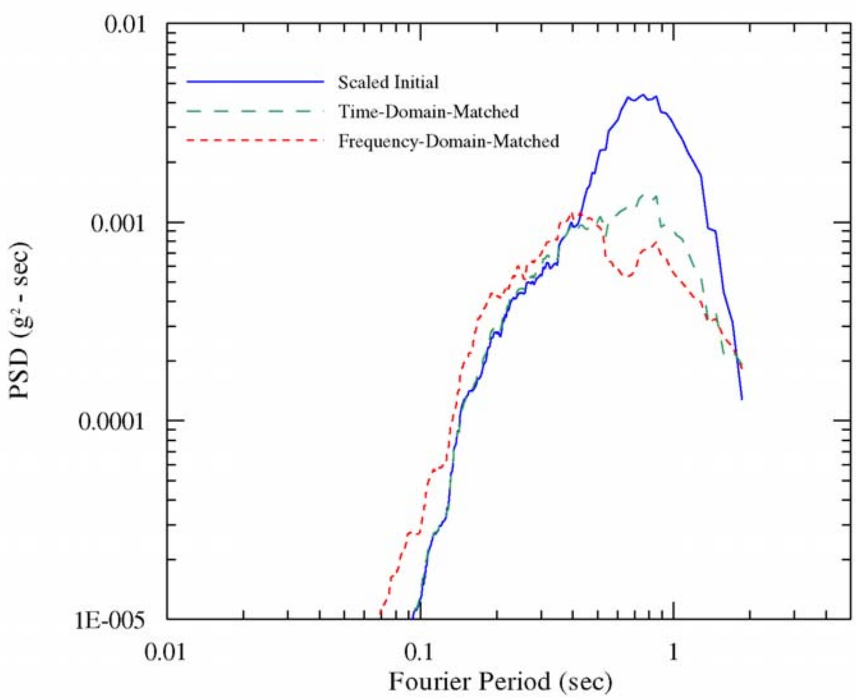


b. Power spectral density functions

Figure C-13. Comparisons of response spectra and power spectral density functions for the scaled Griffith Park recording (component 270E), 1971 San Fernando earthquake, and the spectrum-compatible acceleration time-histories

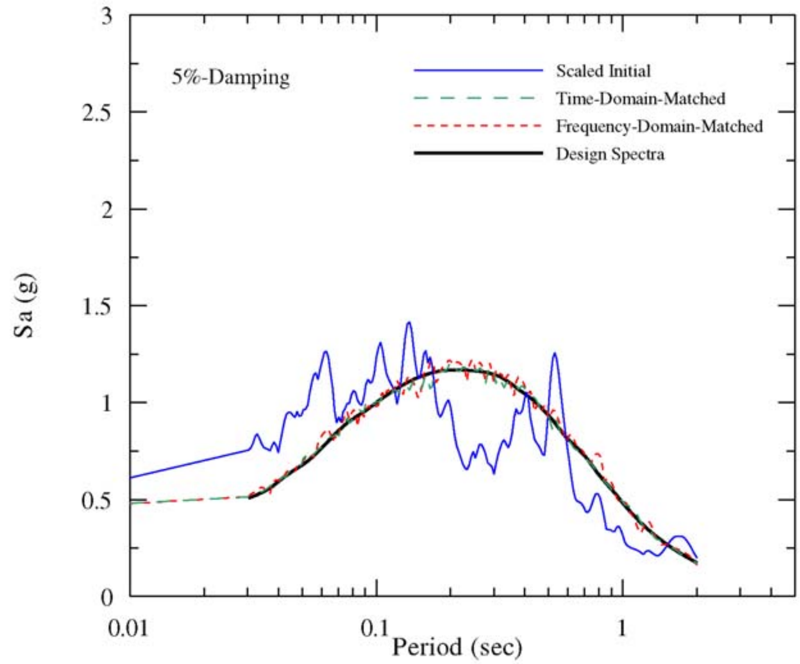


a. 5 percent damped response spectra

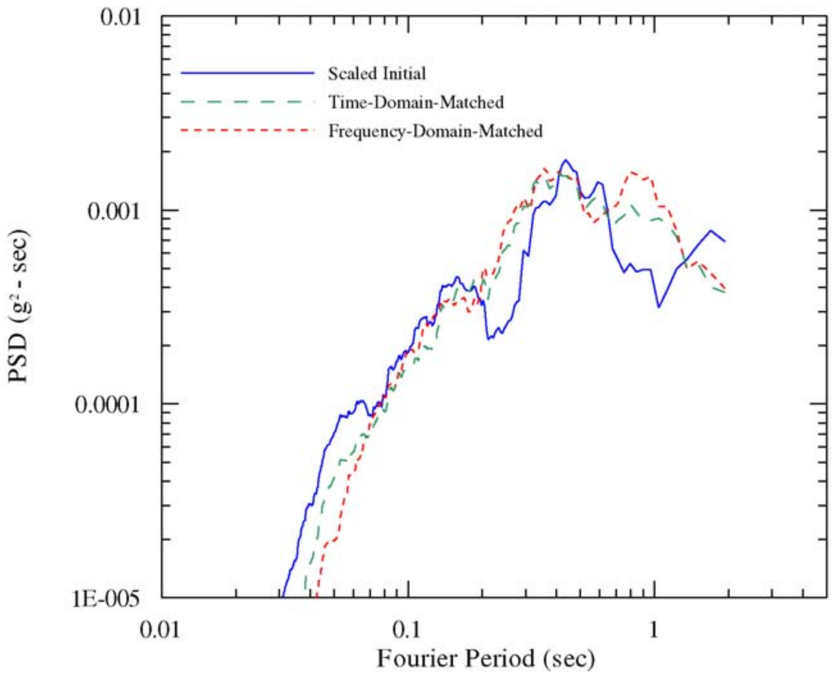


b. Power spectral density functions

Figure C-14. Comparisons of response spectra and power spectral density functions for the scaled Halls Valley recording (component 240E), 1984 Morgan Hill earthquake, and the spectrum-compatible acceleration time-histories



a. 5 percent damped response spectra



b. Power spectral density functions

Figure C-15. Comparisons of response spectra and power spectral density functions for the scaled Gazli recording (component 090E), 1976 Gazli earthquake, and the spectrum-compatible acceleration time-histories

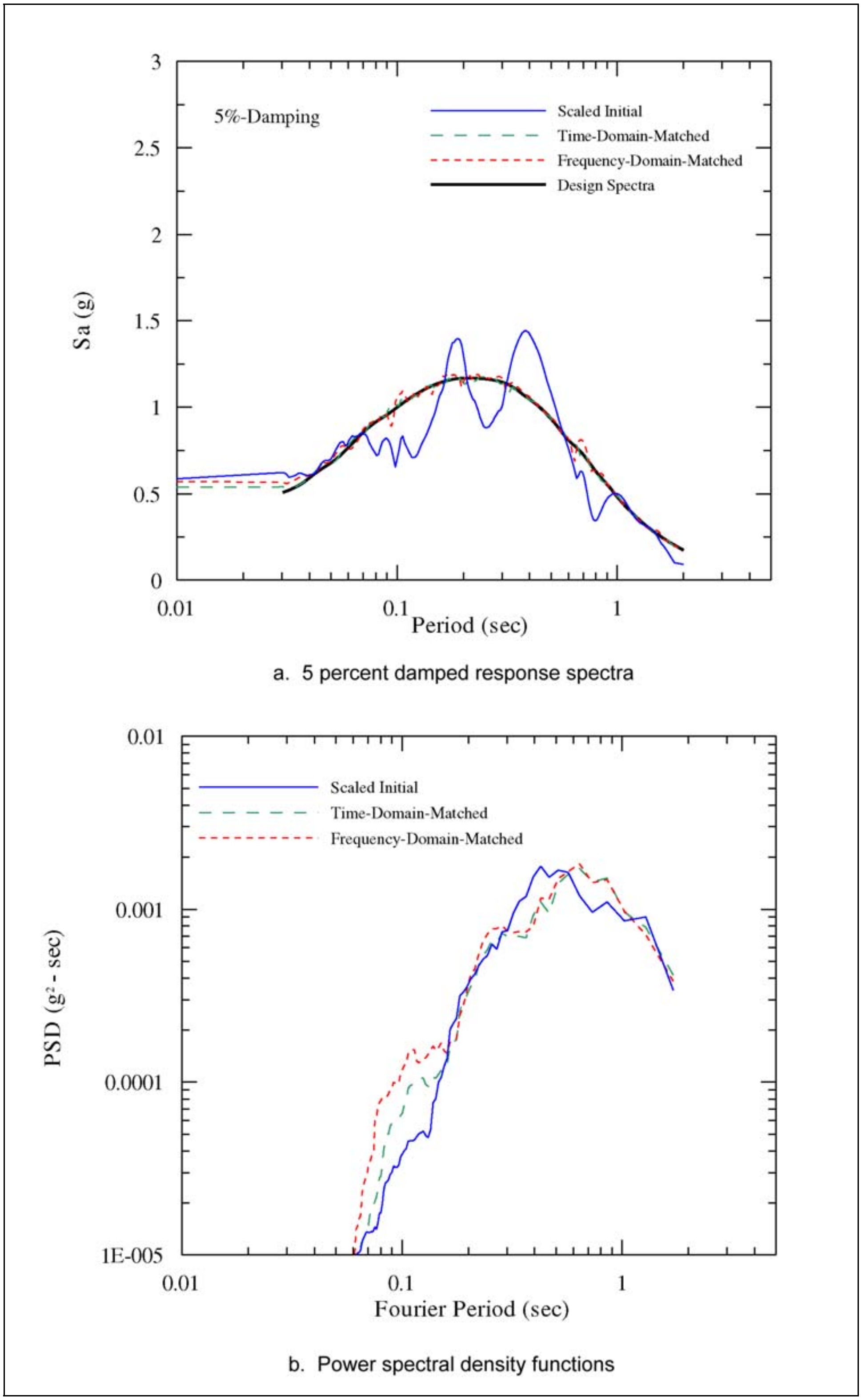
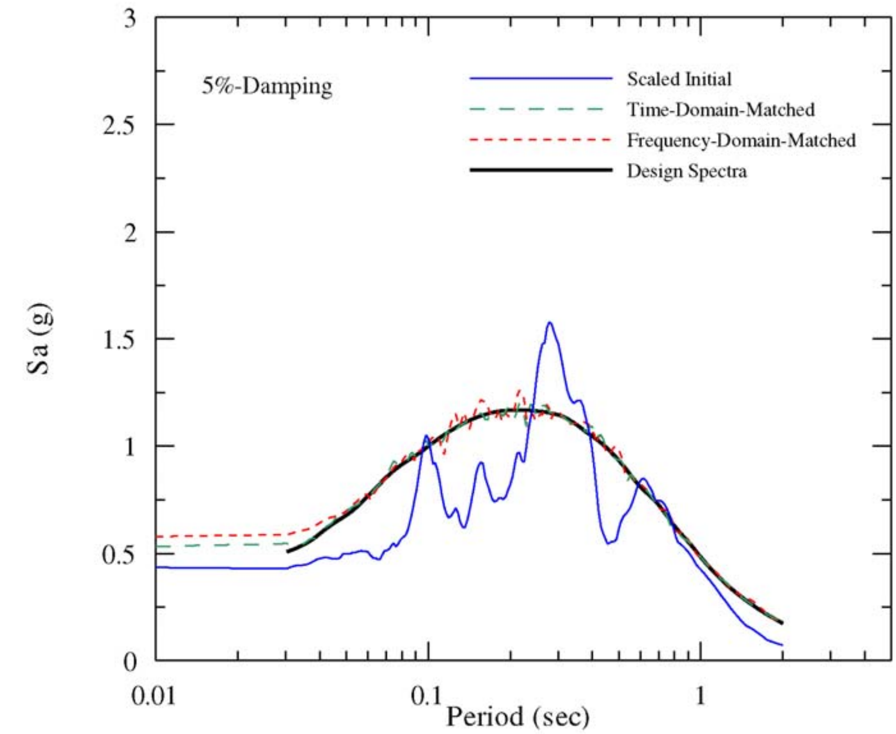
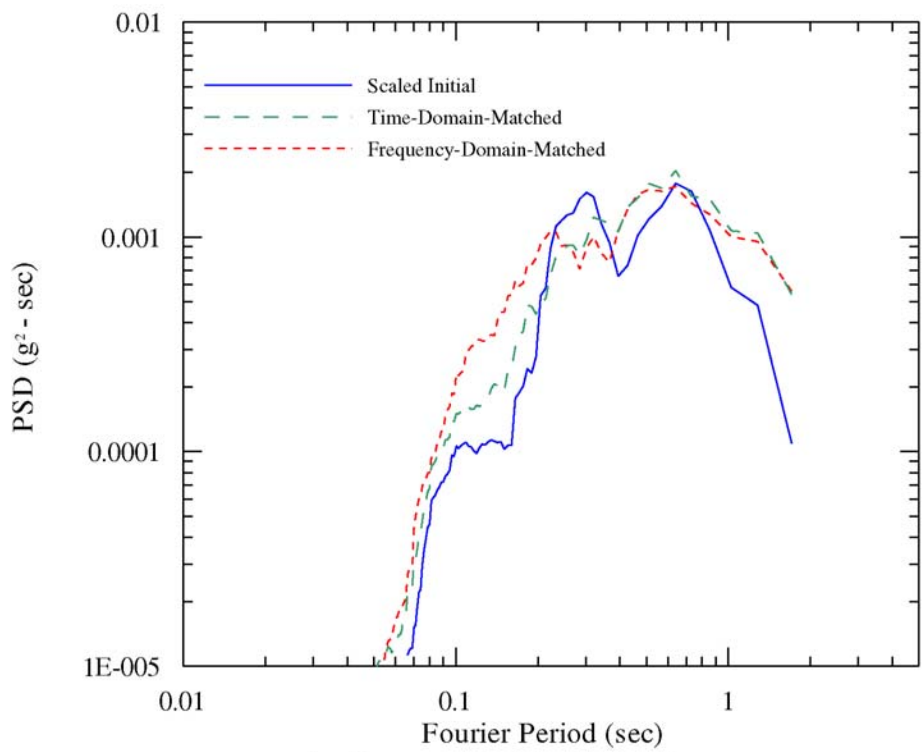


Figure C-16. Comparisons of response spectra and power spectral density functions for the scaled Pacoima Dam recording (component 265E), 1994 Northridge earthquake, and the spectrum-compatible acceleration time-histories

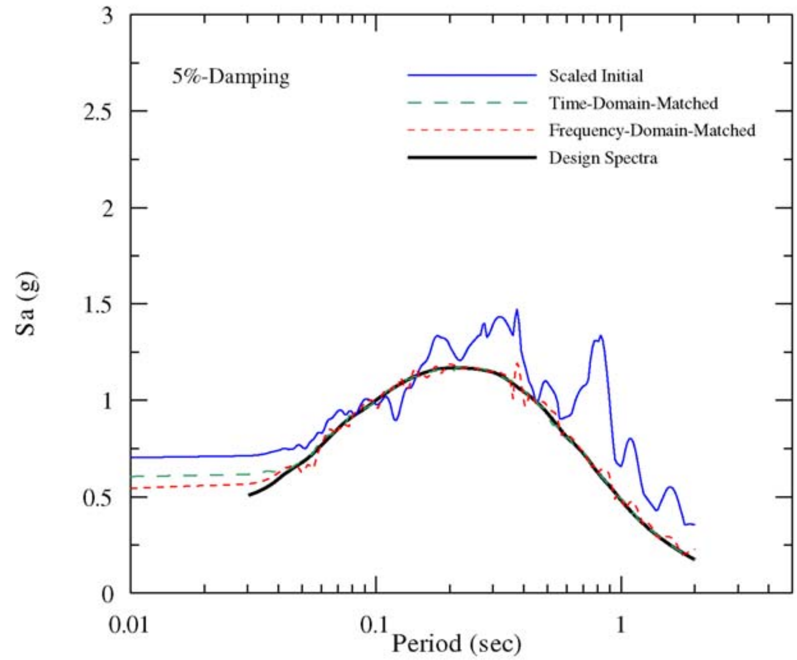


a. 5 percent damped response spectra

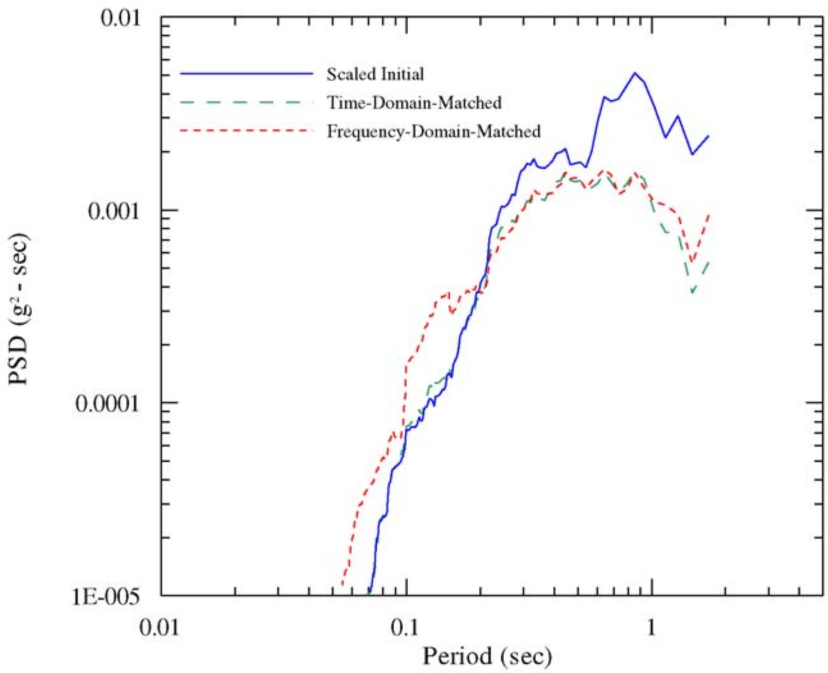


b. Power spectral density functions

Figure C-17. Comparisons of response spectra and power spectral density functions for the scaled Coyote Lake Dam recording (component 195E), 1984 Morgan Hill earthquake, and the spectrum-compatible acceleration time-histories

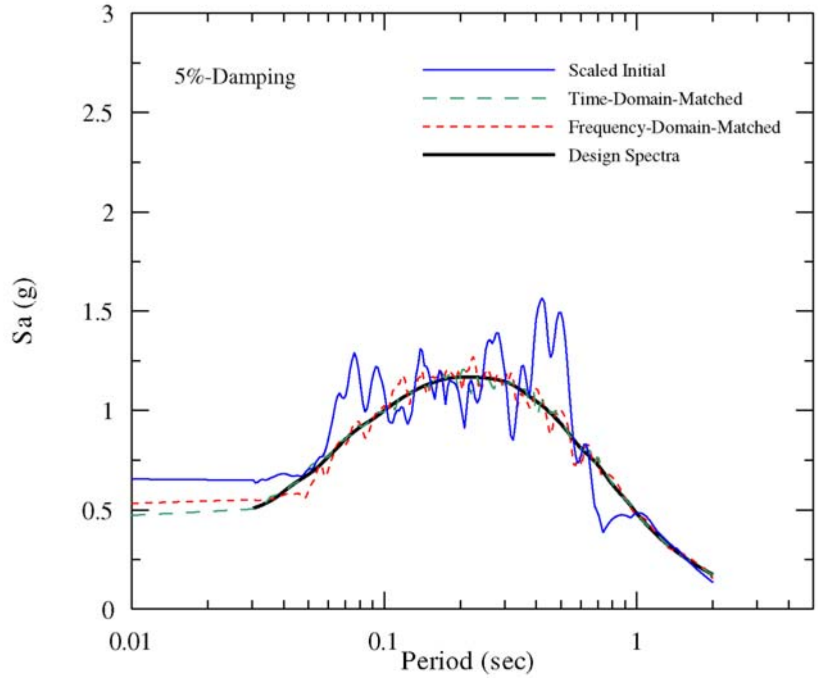


a. 5 percent damped response spectra

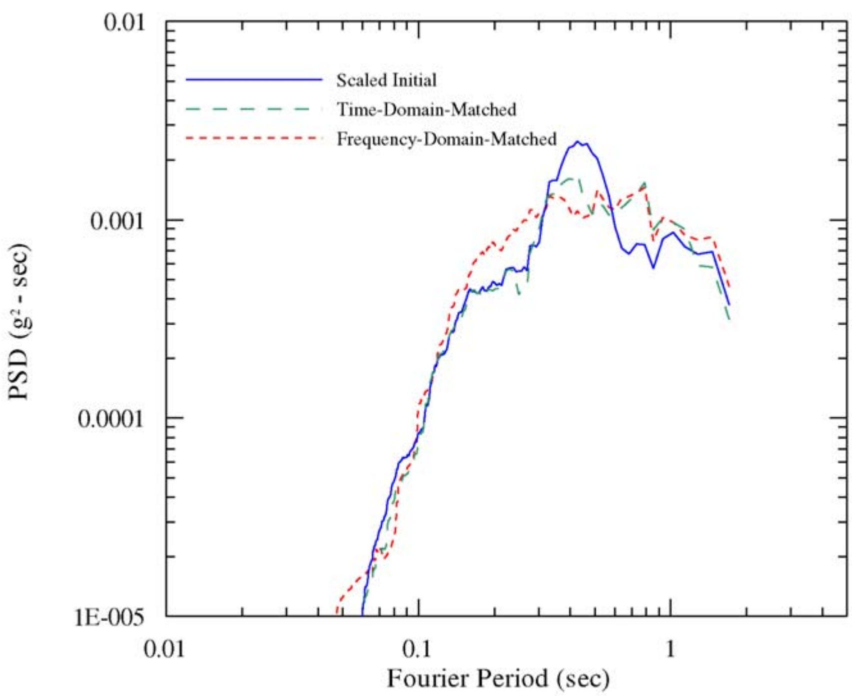


b. Power spectral density functions

Figure C-18. Comparisons of response spectra and power spectral density functions for the scaled SCEC recording (component 018E), 1994 Northridge earthquake, and the spectrum-compatible acceleration time-histories

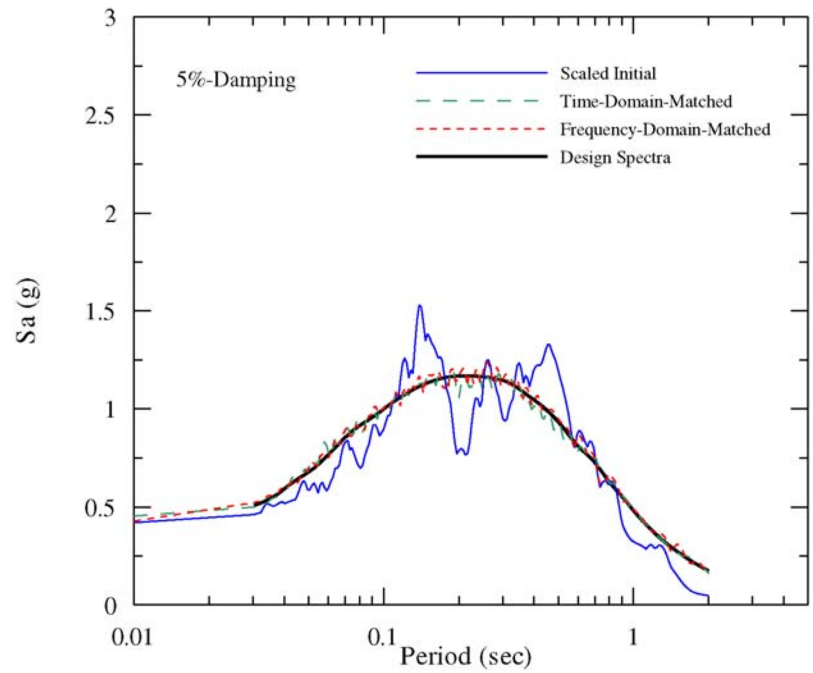


a. 5 percent damped response spectra

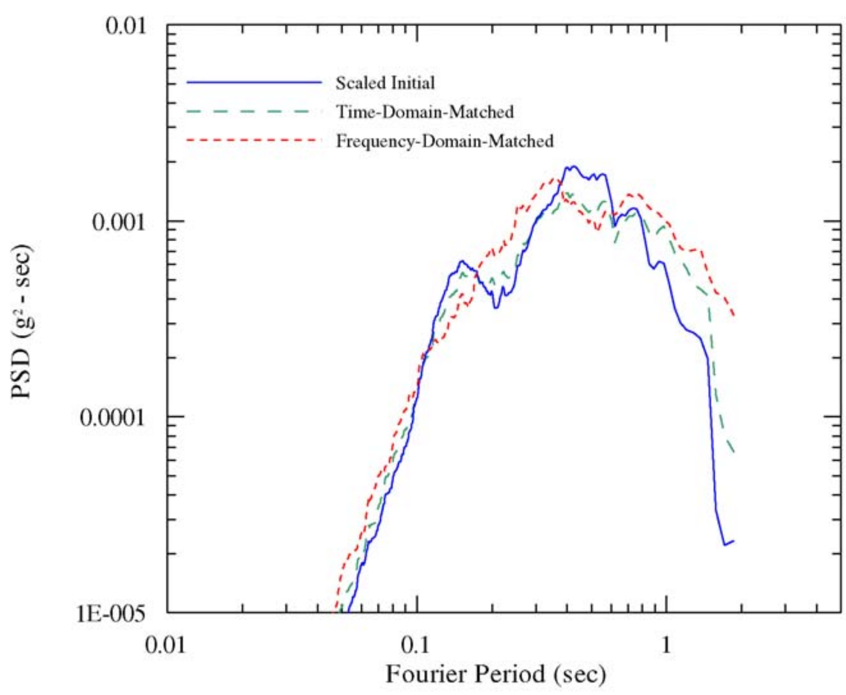


b. Power spectral density functions

Figure C-19. Comparisons of response spectra and power spectral density functions for the scaled Pacoima Dam recording (component 254E), 1971 San Fernando earthquake, and the spectrum-compatible acceleration time-histories



a. 5 percent damped response spectra



b. Power spectral density functions

Figure C-20. Comparisons of response spectra and power spectral density functions for the scaled UCSC BRAN recording (component 090E), 1989 Loma Prieta earthquake, and the spectrum-compatible acceleration time-histories